Engineering Costs and Production Economics, 20 (1990) 165–174
Elsevier

CHANCE CONSTRAINED QUALITY CONTROL

David L. Olson
Department of Business Analysis, Texas A&M University, College Station, Texas 77843, U.S.A.

ABSTRACT

Consideration of quality implicitly introduces the need to adjust inputs in order to obtain desired output. Output evaluation must consider quality as well as cost. Real decisions may involve other objectives as well. Production problems often involve a dynamic situation where the relationship between cost and quality must be experimentally developed. The proposed method is to use regression as a means of identifying input–output relationships, to include variance. A chance constrained multiobjective model can be developed, and decision maker preference incorporated through interactive analysis. Through application of the proposed method, efficient solutions providing as much quality as desired at minimum cost are capable of identification.

Past applications in the area are discussed, as are data collection, modeling, and solution procedures. A number of multiobjective concepts are reviewed in light of the chance constrained model. Constrained techniques are considered more appropriate than weighting techniques for this class of problem. The abilities of currently available solution techniques to support multiobjective analysis for this class of problems are also discussed.

INTRODUCTION

Quality has become a paramount issue in manufacturing. In order to be competitive, it is necessary not only to produce more, but also to ensure that products have desired quality. Traditional means of treating quality with post production sampling are often being replaced by methods which are better at incorporating quality during the production process. Just-in-time manufacturing is an example of this philosophy. Another means to enhance productivity is to design operations in such a way that production output has the desired properties. Examples of management science support to quality in process control are many, including Dalal [1] (steel), Sengupta [2] (paper mill), and Lee and Olson [3] (construction aggregate). Generally, there will be an implicit need for multiobjective analysis to examine the tradeoff between risk (or quality) and product cost. Sengupta applied a goal programming approach to a paper production process, utilizing constraints upon input variables, and optimizing a linear system of statistically derived relationships between inputs and outputs. Dalal utilized simulation for his analysis, comparing a number of alternatives generated by the analyst. Lee and Olson applied nonlinear goal programming to deal with the risk/cost tradeoff. The application of a scientific approach to this problem is demonstrated by Derringer and Suich [4] (rubber compound design), where regression was used to determine the relationship between inputs and output, and optimization of a weighted four objective function was used to design the product. This paper presents a technique for approaching product design, applying the scientific approach of taking experimental results, using regression analysis to determine relationships between inputs and output, and examining the tradeoffs between product consistency and cost. Because estimates of the relationship between inputs and output quality is by nature probabilistic, there is an implicit need to balance multiple objectives (cost and risk). Other objectives, such as use of particular suppliers, emphasis of new materials, and others are also often appropriate.

This paper proposes a general application of the scientific method to quality problems. Very often, especially with new processes, the relationship between inputs and outputs is not strictly known. Standard statistical approaches can be applied to determine this relationship. In some cases, sampling would be appropriate (blending of materials – Lee and Olson [3]). In that case, sampling information provided all information necessary for the design problem. In other applications, it would be appropriate to apply regression analysis [2,4]. In this case, there well may be covariance between inputs. Regardless of whether sampling or regression is used, experimental information can be used as inputs to a multiobjective model that would lead to better support for quality design. A multiple objective model of the quality decision can be made incorporating this information obtained from sampling statistics or regression.

The intent of this paper is to present an overall framework of analysis for the problem of designing quality, using regression or sampling to determine relationships between inputs and outputs, and multiple objective analysis to support the design decision. Multiobjective concepts provide an opportunity to incorporate quality into many production processes and product designs, especially product designs involving varying levels of process controls or mixtures of materials as inputs.

OBJECTIVES IN QUALITY DESIGN

A general approach to obtaining quality would be to obtain the best materials, obtain the best engineering, and apply the best workmanship. This approach, of course, results in very high cost, and therefore, a very limited market. A realistic approach would be to balance available materials and design as well as possible, and put forth the best effort with available workmanship. Product design requires a decision balancing what is available with what the market demands. Minimum cost is not an overriding objective, nor is maximum quality. But good management seeks to obtain as much quality as possible at a reasonable cost. Management also requires consideration of other operational factors.

Consistency is a factor in quality. Consumers expect products they buy to perform as they expect. The market is a clearing house where the demand tradeoff between cost and quality should hold – hopefully better quality is available at higher cost. If products are inconsistent, this detracts from quality to the consumer because there is uncertainty as to the level of quality being purchased. Therefore, quality design is important to producers. There should be a market for *efficient* products, which are appropriately priced for the level of quality in the product.

Producers often have the ability to control product consistency. This is one of the key concepts behind the idea of just-in-time manufacturing. Variance in input materials can be reduced by relying upon a limited number of suppliers, located near the production facility. Using such local materials can be another objective of the decision maker seeking appropriate quality. Further, the just-in-time philosophy increases quality through reduction of product variance by having a limited work in process inventory. This makes it easier to identify production problems, and reduces waste before a great deal of damage is done.

Therefore, product designers have the decision of selecting the relative degree of cost and quality desired in their product. Minimum product cost will not likely provide sufficient quality of output. On the other hand, maximum quality will probably not have a sufficient market either. This may not be a simple two objective problem, but may involve additional objectives as well.

AN APPROACH TO QUALITY DESIGN

The relationship between inputs and outputs is often not perfectly understood. Regression analysis provides a means of establishing this relationship. A scientific approach would be to gather data measuring inputs and outputs, and possibly even experimenting to determine the effect of changing inputs.

Four applications demonstrate the problem. Table 1 presents a recapitulation of the inputs used in these four examples, as well as the responses desired. The first application was presented by Dalal [1], where the decision involved control of a steel hot strip rolling mill. Derringer and Suich [4] sought to meet four quality features for a rubber compound used in tire tread through control of the quantity of three input materials. Sengupta [2] used regression to determine linear (goal) programming coefficients (without utilizing variance information) in a paper mill application seeking better quality output through control of process control levels. All three of these applications used regression to estimate the relationship between inputs and outputs. Another example, Lee and Olson [3], used sampling information in an application to mix a variety of input materials to meet specifications of material size in a construction aggregate blending model. While all four applications involve consideration of quality, and used an approach of first identifying relationships, and then applying a model to aid the decision leading to better quality, they vary significantly in how they proceed. Dalal [1] used simulation, relying upon experimental design to generate

TABLE 1

Examples of regression/sampling to identify input/output relationship

	Control variables	Response variables Objectives
Dalal [1] Steel hot strip mill Simulation	Finishing temperature Gauge of steel Speed of mill Water pressure	Coiling temperature
Derringer and Suich [4] Rubber compound-tire tread Desirability function	Hydrated silica level Silane coupling agent Sulfur level	Abrasion index Modulus Elongation Hardness
Sengupta [2] Paper mill Goal programming model (no variance used)	Cooking temperature Steam pressure Alkali index Sulphidity	Kamyr digester number Burst factor Breaking length
Lee and Olson [3] Aggregate mixture-construction Chance constrained goal programming (sampling information)	Materials of varying quality	Satisfy gradation limits

TABLE 2

Structuring a multiobjective product design model

Determine objectives		
Identify available alternative inputs		
Inputs = {Process controls and/or Alternative materials}		
Determine relationship between inputs and outputs		
Design experiment		
Gather data		
Regress output versus inputs		
Structure model		
Variables	Alternative inputs	
Constraints	Restrictions upon decision Constraints defining objective functions can be nonlinear	
Objectives	Measurable and stated	
Multiobjective analysis		
Generate solutions to gain reference payoff table, or grid of attainment levels		Learning
Interactive generation of improved solutions		Search
Decision maker assessment in light of ALL objectives (stated or not)		Judgement

the alternative decisions that were evaluated. Further, Dalal had one response variable. The other three applications were multiobjective. Derringer and Suich [4] used a nonlinear desirability function, which in effect is the application of a weighted scheme to combine the value of the four response variables in their decision. Sengupta [2] used goal programming to aid the decision involving three response variables. Lee and Olson [3] used chance constrained goal programming to combine objectives of quality, cost, and emphasis of desired material use.

In most business applications, the relationship between inputs and outputs often includes some variance. While averages can and often have been (see [2]) used to estimate impact between inputs and outputs, variance can be an additional important factor in describing the measuring quality. That information is available from regression analysis in many quality control decisions involving production processes. Table 2 presents a framework for the proposed approach.

In general, we can use multiobjective concepts to describe the quality design decision. We would expect decision makers to have a number of objectives, including cost and some measure of quality. There may be other objectives that are expressed as functions, measuring objective attainment. Because there may be other factors, either not well expressed, or possibly not very measurable, *interactive* multiobjective techniques would be attractive. This would allow application of multiobjective analysis without requiring decision makers to fully express their utility function precisely balancing their multiple objectives.

The information obtained through regression analysis can be incorporated into a multiobjective model, applying well developed interactive techniques. The linear programming approach is attractive, in that some function (cost or quality) can be optimized subject to constraints. The tradeoff involved with multiple objective linear programming analyses has been addressed by a number of alternative techniques (Evans [5]; Rosenthal [6]). Optimizing stochastic processes requires solution approaches other than linear programming. Chance constrained programming (CCP) has been developed to optimize systems subject to uncertainty in technological coefficients (Charnes and Cooper [7]). CCP has been applied to many problems where the otherwise linear system involved uncertainty in functional technological coefficients (Hogan et al. [8]). This is appropriate when there is variance in input characteristics and regression output or when sampling can be used to determine the mean and variance of each input to the function.

Regression constraints

The general decision problem is one of controlling the value of a set of input variables $\{X_p\}$ in order to attain appropriate values for one or more (m) response variables $\{Y_j\}$. Each Y_j is assumed to be a function of the set of X_p (Olson et al. [9]):

$$Y_j = f(X_1, X_2, \ldots, X_p) \quad \text{for } j = 1, \ldots, m$$

If this relationship were known with certainty, there was no variance in the relationships, and if there was one clear objective function, the system could be optimized through linear programming. If the functional relationship was nonlinear, variable transformations as used by Dalal [1] or Derringer and Suich [4] could be used. This would yield:

Max Y_j (max $-Y_j$ for a minimization)

subject to: $Y_j = f(X_1, X_2, \cdot X_p)$

any limits on any X_p

This would be ideal, in that the optimal decision (the X values) would be obtained. However, many process relationships ($Y = f(X)$) are unknown, and involve stochastic properties. Sampling data provides a useful means of identifying these stochastic relationships:

$$Y_{ij} = f_i(X_1, X_2, \ldots, X_p) + e_{ij} \quad i = 1, \ldots, n, j = 1, \ldots, m$$

Each of the m relationships can be identified based upon n observations, and while the relationship is expected to be inexact, inferential statistical analyses can be conducted. The error terms for relationships based upon large samples are often normally distributed with mean 0. This results in violation of the linear programming assumption of certainty of the model coefficients. Chance constrained programming techniques have been developed for normally distributed variance of technological coefficients in mathematical programming models involving linear functions (other than the chance constraints). The many applications of chance constrained programming involve stochastic estimates based upon sampling of input variable properties, where the functional relationship between output variables (Y_j) and input variables (X_p) involves normally distributed errors from a linear function. This form of function is often appropriate where sampling is the basis of determining the relationship. This yields a general form:

Max Y_j

subject to: $Y_j = f_j(X_1, X_2, \ldots, X_p) \pm z(V)$

$\Pr\{Y_j \leqslant b_k\} \geqslant \alpha$ confidence level

any limits on X_p

where z is the normal distribution functional and V is the variance–covariance matrix of the input variables X_p. Chance constraints operate through penalty functions ($z(V)$) which provide greater than 0.5 probability that the functional limits will be met. Y_j can be replaced with

$$\beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_p X_p \pm z(V)$$

where the matrix $(\mathrm{V}) = [\beta][\mathrm{X}'\mathrm{X}]^{-1}$. Chance constraints introduce the additional decision problem of setting the desired probability level of attainment. The decision problem is more complex, in that the tradeoff between objectives, such as cost and quality probability must be addressed. However, this provides a more comprehensive treatment of the process control decision problem.

The introduction of risk levels makes the decision problem a tradeoff between cost (or profit) and probability of chance constraint attainment. While solution of chance constrained models is more involved than linear programming models, a number of solution techniques have been presented (see solution technique references). When these relationships are based upon general linear regression, there are additional forms of error introduced by the estimator of the intercept.

That is, the general linear model is of the form:

$$Y'_j = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + e$$

The impact of the error in estimation of β_0 modifies the penalty function for chance constraints (V):

$$(V) = [\beta]\mathrm{mse}[1 + X'X]^{-1}$$

Chance constrained models can still be applied to the optimization model, with the (V) element including additional information. This can be solved by the same techniques as the commonly used chance constrained form, through modification of input data (an additional X variable, held constant at value 1, is included in the model, providing a means to input the constant term in the variance covariance matrix (V)).

Chance constrained model forms

Consideration of chance constraints introduces the need to consider probabilistic forms as objectives. Charnes and Cooper [10] presented three formulations to incorporate chance constraints into mathematical programming models. One approach would be to maximize the expected value of the probabilistic function (their 'E' model):

Max $E[Y]$ (where $Y = f(X)$)

Subject to: $\Pr\{Ax \leqslant b\} \geqslant \alpha$

Any coefficient of this model (Y, A, b) may be probabilistic. The intent of this formulation would be to maximize (or minimize) a function while assuring α probability that a constraint is met. Note that the expected value of a function usually involves a linear functional form, although the chance constraint will likely be nonlinear. This form would be appropriate in many blending problems, and many such applications have been presented.

Another form would be to minimize variance ('V' model):

Min $\mathrm{Var}[Y]$

Subject to: $\Pr\{Ax \leqslant b\} \geqslant \alpha$

Minimizing variance, as noted before, may have some utility in quality control, although the intent is usually to accomplish some functional performance level while satisfying the chance constraint set. Incorporating this class of objective would be similar to that of incorporating the E model concept.

The third form of Charnes and Cooper would be to maximize the probability of satisfying a chance constraint set ('P' model):

Max $\Pr\{Y \geqslant \text{target}\}$

Subject to: $\Pr\{Ax \leqslant b\} \geqslant \alpha$

This form is generally much more difficult to accomplish in a mathematical programming analysis, especially if there are joint chance constraints. The only practical means to accomplish this would be to run a series of models, seeking the highest α level yielding a feasible solution.

Most applications of chance constrained models assume normal distributions for model coefficients. Goicoechea and Duckstein [11] presented deterministic equivalents for non-normal distributions which can be incorporated into chance constrained models. In general, chance constrained models become much more difficult to analyze if the variance of parameter estimates increases. While it is not difficult to model a negative exponential distribution for a model coefficient, the probability of satisfying a chance constraint with such distributions is much lower.

Many chance constrained applications also assume coefficient independence. This is often appropriate. However, the covariance elements of coefficient estimates can easily be incorporated as well, eliminating the need to assume coefficient independence. When considering chance constraints developed from regression analysis, independence of coefficient estimates can be obtained by experimental design. However, data must often be obtained without the means of imposing an ideal experimental design plan. Covariance terms of regression estimates can be used in chance constrained models without difficulty.

Solution methods for chance constrained models

A number of solution techniques (with codes) have been published, Seppala and Orpana [12] reported the relative efficiency of Seppala's nonlinear code, which is based upon piecewise linear approximation. This code is capable of dealing with any size of model suitable for linear programming analysis, and is efficient because it focuses break points for the piecewise linear approximation as the algorithm progresses toward solution. Lee and Olson [13] reported a gradient algorithm incorporating preemptive goal programming. By setting one preemptive level, and applying weights to deviational variables to be minimized, other forms of multiobjective analysis can be applied as well as preemptive goal programming. This code is more limited by model size than Seppala's piecewise linear code. Weintraub and Vera [14] presented a cutting plane chance constrained code which has been applied to very large forestry models. Additionally, Rees et al. [15] presented response surface methodology as a means to analyze multiobjective nonlinear problems. There also exist approximation techniques which could be incorporated in linear programming models (Hillier [16]; Olson and Swenseth [17]), although approximations involve a tradeoff between computational effort and accuracy.

Another approach would be to utilize newly marketed nonlinear codes. For instance, LINDO has a quadratic programming capability, although the sister product GINO provides much stronger nonlinear model support. Other mathematical programming packages are also being published with very strong nonlinear capability.

MULTIOBJECTIVE CONCEPTS APPLIED TO PROBABILISTIC PROGRAMMING

Key concepts in multiobjective analysis have been well developed. A recent review of these concepts was provided by Michalowski [18]. A number of problem categories can be identified. If well developed knowledge of the problem is known, to include a utility function, the problem of multiple objectives can be dealt with as a well structured mathematical model, optimizing utility subject to known constraints. However, in practice utility functions are rarely well developed. An alternative approach would be to *generate all nondominated solutions*. In an entirely linear model, this is accomplished through identification of nondominated extreme points, realizing that there is a nondominated surface connecting these extreme points. One difficulty with this approach is that there may be a very large number of such nondominated solutions. If *a priori* knowledge of decision maker preferences is available, the nondominated solution best reflecting this preference structure can be identified. There are two well developed approaches: use of *weights to combine the multiple objectives*, or using *constraints upon objective attainment* to generate solutions. This can be done in aggregate, where the weights imply relative importance, or lexicographically. However, *a priori* analysis often assumes more perfect knowledge of decision maker preference structure than is merited in practice. At least two approaches to support problems where less than perfect knowledge of decision maker preference structure have been developed. Goal programming allows incorporation of aspiration levels for various objectives through target levels, either in a minimization of weighted deviations or preemptive form. Compromise programming seeks to identify solutions minimizing some metric from an ideal (but infeasible) solution. *Interactive* approaches seek to utilize decision maker selection from a subset of nondominated solutions to identify the solution representing the best tradeoff among objectives. Interactive approaches allow greater support to decision maker learning. A key idea in this approach is often the development of a *payoff table* among the various objectives, giving a decision maker a reference for relative attainment on each objective.

A general formulation of multiobjective models would be:

Max $f_k(x)$ for $k = 1,...,K$ objectives

Subject to: $f_i(x) \leqslant b_i$ for $i = 1,...,M$ constraints

Weighted methods use the objective function:

Max $\sum w_k f_k(x)$

Subject to: $f_i(x) \leqslant b_i$ for required constraints

$\sum w_k = 1$ (optional)

Preemptive goal programming models have the objective function:

Min $P_1\{w_1\, d_1 \pm\};...; P_k\{w_k\, d_i \pm\}$

Subject to: $f_i(x) + d_i^- - d_i^+ = b_i$

where $P_i >>> P_2 >>> ... >>> P_k$

Constraints upon objective attainments can be imposed in order to generate nondominated solutions.

Max $f_j(x)$ where j is a selected objective

Subject to: $f_i(k) \leqslant b_i$ for required constraints

$f_k(x) \geqslant$ target for all objectives except objective j

In this approach, the targets for constrained objectives can be varied as selected by the decision maker. This operates very much the same as the goal programming model. The benefit of using constraints upon objectives in analyzing multiobjective chance constrained models is that a more controlled grid of objective function attainment levels can be obtained.

All of these multiobjective programming concepts can be applied to modeling probabilistic constraints. However, there are varying complications involved due to the nonlinear form of probabilistic constraints.

Generation of nondominated solutions

Development of a chance constrained model leads to a nonlinear efficient frontier if a nonlinear constraint is binding in the direction of search. This complicates the generation of the nondominated set. While the nondominated set in an entirely linear model is also liable to be infinitely large, each nondominated point can be described as a combination of the finite set of nondominated extreme points. This is no longer true in a nonlinear model. Rakes and Reeves [19] presented a methodology which would transform the nonlinear objective into a linear form, thus allowing generation of all nondominated extreme points. However, in general, two problems exist with the idea of generating nondominated sets in multiobjective chance constrained models. First, transforming to a linear function may not be convenient or match decision maker style. Second, even for linear models, there may be a very large number of nondominated extreme points. Therefore, in order to provide decision makers with a reference of tradeoffs, only a subset of the nondominated set will be practical.

Weights versus constraints

There are two general approaches used in multiobjective programming to generate nondominated solutions. Approaches such as Steuer's Method and the Method of Zionts and Wallenius rely upon relative weights for each objective function. Other methods, such as the Step Method and goal programming, operate by generating nondominated solutions through bounding objective attainment. While the use of the weighted approach is certainly feasible, consideration of probabilities is difficult to coordinate with other objective measures, such as profit or cost (Rakes and Franz [20]). Therefore, the weights will have little meaning by themselves unless effort is taken to eliminate differences in scale. Use of bounds upon objective function attainment would seem to serve a more general purpose. Furthermore, considering the difficulty of using the 'P' form (maximizing probability) as an objective function, it is much more practical to measure the probability of a constraint being satisfied once a solution is obtained than maximizing probability directly as an objective.

Interactive approach

The spirit of decision making under conditions of multiple objectives is better served by interactive approaches, allowing reflection of decision maker learning, as well as allowing decision makers to judge the relative accomplishments of all objectives. Further, decision makers may not be able to express all features of the decision that they consider. Therefore, *a priori* approaches may not be appropriate in the design problem.

An interactive approach would better support the learning process of the decision maker if an initial reference set of feasible objective attainment levels were provided. This can be accomplished by following the idea of a payoff table, as developed in the Step Method. Generation of this payoff table in a chance constrained model would be little different from the procedure used in linear multiobjective models, although it may require additional effort to generate solutions maximizing probability of attainment. While weighted procedures can be used, it is easier to control attainments that the decision maker may specify by use of constraint techniques.

CONCLUSIONS

Product quality is a crucial element of contemporary manufacturing. The quality designed into a product requires consideration of multiple objectives. In some product design decisions, such as those blending materials, or those involving control of some process, a key element of quality is the consistency of outputs. The design problem may involve relationships between inputs and outputs that are unknown.

A framework for approaching this product or process design was presented. It suggests incorporation of an overall scientific approach with at least three stages. Stage 1 would be application of the scientific method to experiment (or use available data) to determine the relationship between inputs and outputs. Regression analysis would be a valuable tool to support this phase. Stage 2 would be to structure the problem into a multiobjective model. Elements of this stage include establishing measurable objectives, identifying constraints to the decision (including incorporating chance constraints obtained from regression), and obtaining reference objective attainment levels through generation of a representative subset of nondominated solutions. The last stage would be to apply interactive multiobjective analysis to seek a solution useful to the decision maker. Interactive analysis is considered appropriate, because all objectives may not be expressible in measurable terms, and the decision maker would be expected to be learning from the analysis.

The complications involved with solution algorithms was discussed. Chance constrained models require nonlinear programming support. This may preclude direct application of some multiobjective techniques relying upon specialized computer codes. However, general concepts can still be applied. Complications arise when considering nondominated solutions, because many multiobjective techniques rely upon the ability to generate extreme points, which is more complicated in nonlinear models. Further, reliance upon relative weights for each objective is complicated, due to the potentially high relative impact of minor changes in weights. Use of constraints to control identification of objective tradeoffs is recommended for chance constrained multiobjective models.

REFERENCES

1 Dalal, J.G., 1978. Evaluation of a coiling temperature-control procedure for a hot strip mill. Journal of Quality Technology, 10: 31–39.

2 Sengupta, S., 1981. Goal programming approach to a type of quality control problem. J. Oper. Res. Soc., 32: 207–211.

3 Lee, S.M. and Olson, D.L., 1983. Chance constrained aggregate blending. J. Constr. Eng. Manage., 109 (1): 39–47.

4 Derringer, G. and Suich, R., 1980. Simultaneous optimization of several response variables. J. Qual. Technol., 12 (4): 214–19.
5 Evans, G.W., 1984. An overview of techniques for solving multiobjective mathematical programming problems. Manage. Sci., 30 (11): 1268–1282.
6 Rosenthal, R.E., 1985. Principles of multiobjective optimization. Decison Sci., 16 (2): 133–152.
7 Charnes, A. and Cooper, W.W., 1959. Chance constrained programming. Manage. Sci., 6 (1): 73–79.
8 Hogan, A.J., Morris, J.G. and Thompson, H.E., 1981. Decision problems under risk and chance constrained programming. Manage. Sci., 27 (6): 698–716.
9 Olson, D.L., Wei, J.C. and White, E.M., 1988. Prediction-interval constrained programming. J. Qual. Technol., 20 (2): 90–97.
10 Charnes, A. and Cooper, W.W., 1963. Deterministic equivalents for optimizing and satisficing under chance constraints. Oper. Res., 11: 18–39.
11 Goicoechea, A. and Duckstein, L., 1987. Nonnormal deterministic equivalents and a transformation in stochastic mathematical programming. Appl. Math. and Computation, 21(1): 51–72.
12 Seppala, Y. and Orpana, T., 1984. Experimental study on the efficiency and accuracy of a chance-constrained programming algorithm. Eur. J. Oper. Res., 16: 345–357.
13 Lee, S.M. and Olson, D.L., 1985. A gradient algorithm for chance constrained nonlinear goal programming. Eur. J. Oper. Res., 22: 359–369.
14 Weintraub, A. and Vera, J., 1988. A cutting plane approach for chance constrained linear programs. Working paper, Dept. of Ind. Eng., University of Chile.
15 Rees, L.P., Clayton, E.R. and Taylor, B.W. III, 1985. Solving multiple response simulation models in using modified response surface methodology within a lexicographic goal programming framework. IIE Trans., 17 (1): 47–57.
16 Hillier, F.S., 1967. Chance constrained programming with 0–1 or bounded continuous decision variables. Manage. Sci., 14: 34–57.
17 Olson, D.L. and Swenseth, S.R., 1987. A linear approximation for chance constrained programming. J. Oper. Res. Soc., 38 (3): 261–267.
18 Michalowski, W., 1988. Use of the displaced worst compromise in interactive multiobjective programming. IEEE Trans. Syst. Man. Cybern., 18(3): 472–477.
19 Rakes, T.R. and Reeves, G.P., 1985. Selecting tolerances in chance-constrained programming: A multiple objective linear programming approach. Oper. Res. Letters, 22: 65–69.
20 Rakes, T.R. and Franz, L.S., 1985. Interpreting goal attainment in chance-constrained goal programming. Omega, 13(1): 73–74.

(Received August 1989; accepted in revised form May 1990)