The Method of Grey Related Analysis to Multiple Attribute Decision Making Problems with Interval Numbers

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Abstract—Multiple attribute decision making is important in many decision making contexts where tradeoffs are involved. The use of qualitative input has proven especially attractive, allowing subjective inputs to be used. However, such systems inherently involve uncertainty with respect to parameter inputs, especially when multiple decision makers are involved. This paper presents the method of grey related analysis to this problem, using interval fuzzy numbers. The method standardizes inputs through norms of interval number vectors. Interval valued indexes are used to apply multiplicative operations over interval numbers. The method is demonstrated on a practical problem. © 2005 Elsevier Science Ltd. All rights reserved.

1. INTRODUCTION

In the past 20 years there has been a great deal published concerning decision theory and multiattribute decision making. This research activity has spanned decision science, system engineering, management science, operations research, and many practical fields of application. Contemporary decision making is conducted in a highly dynamic environment, involving complex tradeoffs

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and high levels of uncertainty. Practical decision problems involve uncertainty with respect to all elements of the basic decision making model (relative attribute weights by decision maker, index values of how well available attributes are expected to perform on each of these attributes [1]).

The uncertainty and fuzziness inherent in decision making makes the use of precise numbers problematic in multiattribute models. Decision makers are usually more comfortable providing intervals for specific model input parameters. Interval input in multiattribute decision making has been a very active field of research. Methods applying intervals have included,

(1.) use of interval numbers as the basis for ranking alternatives [2–8],
(2.) error analysis with interval numbers [9,10],
(3.) use of linear programming and object programming with feasible regions bounded by interval numbers [11–14],
(4.) use of interval number ideal alternatives to rank alternatives by their nearness to the ideal [15].

This paper will present the method of grey related analysis as a means to reflect uncertainty in multiattribute models through interval numbers. Grey system theory was developed by Deng [16], based upon the concept that information is sometimes incomplete or unknown. The intent is the same as with factor analysis, cluster analysis, and discriminant analysis, except that those methods often don't work well when sample size is small and sample distribution is unknown [17]. With grey related analysis, interval numbers are standardized through norms, which allow transformation of index values through product operations. The method is simple, practical, and demands less precise information than other methods. Grey related analysis and TOPSIS [18–20], both use the idea of minimizing a distance function. However, grey related analysis reflects a form of fuzzification of inputs, and uses different calculations, to include different calculation of norms. Feng and Wang [21], applied grey relation analysis to select representative criteria among a large set of available choices, and then used TOPSIS for outranking.

Fuzzy Multiple Criteria Analysis

AHP was presented [22] as a way to take subjective human inputs in a hierarchy and convert these to a value function. This method has proven extremely popular. Saaty used the eigenvector approach to reconcile inconsistent subjective inputs. A number of issues concerning rank reversal [23] and other problems of computational stability have been discussed in the literature ([24] as one of many). Barzilai and Golany [25], proposed the geometric mean of inputs instead, and Lootsma proposed a different scaling method in his REMBRANDT system [26]. Salo and Hamalainen [27], published their interval method using linear programming over the constrained space of weights and values as a means to incorporate uncertainty in decision maker inputs to AHP hierarchies.

The problem of synthesizing ratio judgments in groups was considered very early in AHP [28], and the geometric mean was found to provide satisfactory properties. Fuzzy AHP was proposed as another way to reflect uncertainty in subjective inputs to AHP in the same group context [29–31]. Simulation has been presented as a way to rank order alternatives in the context of AHP values and weights [32].

Other multiple criteria methods besides AHP have considered fuzzy input parameters. ELEC-TRE [33] and PROMETHEE [34] have always allowed fuzzy input for weights. A multiattribute method involving fuzzy assessment for selection has been given in the airline safety domain [35] and for multiple criteria selection of employees [36]. Sensitivity in multiattribute models with fuzzy inputs [37], as well as goal programming [38]. Rough set applications have also been presented [39].

The Method of Grey Related Analysis

Grey related analysis has been used in a number of applications. In our discussion, we shall use the concept of the norm of an interval number column vector, the distance between intervals, product operations, and number-product operations of interval numbers.

Let $a = [a^-, a^+] = \{x \mid a^- \leq x \leq a^+,\ a^- \leq a^+,\ a^-,\ a^+ \in R\}$. We call $a = [a^-, a^+]$ an interval number. If $0 \leq a^- \leq a^+$, we call interval number $a = [a^-, a^+]$ a positive interval number. Let $X = ([a_1^-, a_1^+], [a_2^-, a_2^+], \ldots, [a_n^-, a_n^+])^\top$ be an n-dimension interval number column vector.

DEFINITION 1. *If $X = ([a_1^-, a_1^+], [a_2^-, a_2^+], \ldots, [a_n^-, a_n^+])^\top$ is an arbitrary interval number column vector, the norm of X is defined here as,*

$$\|X\| = \max\left(\max\left(\left|a_1^-\right|, \left|a_1^+\right|\right), \max\left(\left|a_2^-\right|, \left|a_2^+\right|\right), \ldots, \max\left(\left|a_n^-\right|, \left|a_n^+\right|\right)\right).$$

DEFINITION 2. *If $a = [a^-, a^+]$ and $b = [b^-, b^+]$ are two arbitrary interval numbers, the distance from $a = [a^-, a^+]$ to $b = [b^-, b^+]$,*

$$|a - b| = \max\left(\left|a^- - b^-\right|, \left|a^+ - b^+\right|\right).$$

DEFINITION 3. *If k is an arbitrary positive real number, and $a = [a^-, a^+]$ is an arbitrary interval number, then $k \cdot [a^-, a^+] = [ka^-, ka^+]$ will be called the number product between k and $a = [a^-, a^+]$.*

DEFINITION 4. *If $a = [a^-, a^+]$ is an arbitrary interval number, and $b = [b^-, b^+]$ are arbitrary interval numbers, we shall define the interval number product $[a^-, a^+] \cdot [b^-, b^+]$ as follows,*

(1) *when $b^+ > 0$ $[a^-, a^+] \cdot [b^-, b^+] = [a^- b^-, a^+ b^+]$,*
(2) *when $b^+ < 0$ $[a^-, a^+] \cdot [b^-, b^+] = [a^+ b^-, a^- b^+]$.*

If $b^+ = 0$, the interval reverts to a point, and thus we would return to the basic crisp model.

2. PRINCIPLE AND STEPS OF THE METHOD OF GREY RELATED ANALYSIS TO MULTIPLE ATTRIBUTE DECISION MAKING PROBLEM WITH INTERVAL NUMBERS

In our discussion, suppose that multiple attribute decision making problem with interval numbers has m feasible alternatives X_1, X_2, $\ldots$, X_m, n indexes, the weight value w_j of index G_j is uncertain, but we know that $w_j \in [c_j, d_j]$. Here, $0 \leq c_j \leq d_j \leq 1$, $j = 1, 2, \ldots, n$, $w_1 + w_2 + \cdots + w_n = 1$, the index value of jth index G_j of feasible alternative X_i is an interval number $[a_{ij}^-, a_{ij}^+]$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$. When $c_j = d_j$, $j = 1, 2, \ldots, n$, the multiple attribute decision making problem with interval numbers is an interval valued multiple attribute decision making problem with crisp weights. When $a_{ij}^- = a_{ij}^+$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$, the alternative scores over criteria are crisp. The principle and steps of this method are given below.

STEP 1: CONSTRUCT DECISION MATRIX A WITH INDEX NUMBER OF INTERVAL NUMBERS.

If the index value of jth index G_j of feasible alternative X_i is an interval number $[a_{ij}^-, a_{ij}^+]$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$, decision matrix A with index number of interval numbers is defined as follows,

$$A = \begin{bmatrix} [a_{11}^-,\ a_{11}^+] & [a_{12}^-, a_{12}^+] & \ldots & [a_{1n}^-, a_{1n}^+] \\ [a_{21}^-, a_{21}^+] & [a_{22}^-, a_{22}^+] & \ldots & [a_{2n}^-, a_{2n}^+] \\ \ldots & \ldots & \ldots & \ldots \\ [a_{m1}^-, a_{m1}^+] & [a_{m2}^-, a_{m2}^+] & \ldots & [a_{mn}^-, a_{mn}^+] \end{bmatrix}$$

STEP 2: TRANSFORM "CONTRARY INDEX" INTO POSITIVE INDEX.

The index is called a positive index if a greater index value is better. The index is called a contrary index if a smaller index value is better. We may transform contrary index into positive index if j^{th} index G_j is contrary index

$$\left[b_{ij}^-, b_{ij}^+\right] = \left[-a_{ij}^+, -a_{ij}^-\right] \qquad i = 1, 2, \ldots, m.$$

Without loss of generality, in the following, we supposed that all the indexes are "positive indexes".

STEP 3: STANDARDIZE DECISION MATRIX A WITH INDEX NUMBER OF INTERVAL NUMBERS TO GAIN STANDARDIZING DECISION MATRIX $R = [r_{ij}^-, r_{ij}^+]$.

If we mark the column vectors of decision matrix A with interval-valued indexes with A_1, A_2, ..., A_n the element of standardizing decision matrix $R = [r_{ij}^-, r_{ij}^+]$ is defined as the following,

$$\left[r_{ij}^-, r_{ij}^+\right] = \left[\frac{a_{ij}^-}{\|A_j\|}, \frac{a_{ij}^+}{\|A_j\|}\right], \qquad i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n.$$

Note that TOPSIS uses the root mean square to evaluate distance. Grey related analysis uses a different norm, based on minimization of maximum distance.

STEP 4: CALCULATE INTERVAL NUMBER WEIGHTED MATRIX $C = ([c_{ij}^-, c_{ij}^+])_{m \times n}$.

The formula for calculation of the interval number weighted matrix $C = ([c_{ij}^-, c_{ij}^+])_{m \times n}$ is,

$$\left[c_{ij}^-, c_{ij}^+\right] = [c_j, d_j] \cdot \left[r_{ij}^-, r_{ij}^+\right], \qquad i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n.$$

STEP 5: DETERMINE REFERENCE NUMBER SEQUENCE.

The vector for the reference number sequence is determined as the set of optimal weighted interval values associated with each of the n attributes. $U_0 = ([u_0^-(1), u_0^+(1)], [u_0^-(2), u_0^+(2)], \ldots, [u_0^-(n), u_0^+(n)])$ is called a reference number sequence if $u_0^-(j) = \max_{1 \le i \le m} c_{ij}^- \ u_0^+(j) = \max_{1 \le i \le m} c_{ij}^+$, $j = 1, 2, \ldots, n$.

STEP 6: CALCULATE THE CONNECTION BETWEEN THE SEQUENCE COMPOSED OF WEIGHT INTERVAL NUMBER STANDARDIZING INDEX VALUE OF EVERY ALTERNATIVE AND REFERENCE NUMBER SEQUENCE.

The connection coefficient $\xi_i(k)$, between the sequence composed of weight interval number standardizing index value of every alternative $U_i = ([c_{i1}^-, c_{i1}^+], [c_{i2}^-, c_{i2}^+], \ldots, [c_{in}^-, c_{in}^+])$ and reference number sequence $U_0 = ([u_0^-(1), u_0^+(1)], [u_0^-(2), u_0^+(2)], \ldots, [u_0^-(n), u_0^+(n)])$ is calculated. The formula of $\xi_i(k)$ is,

$$\xi_i(k) = \frac{\min\limits_i \min\limits_k \left|\left[u_0^-(k), u_0^+(k)\right] - \left[c_{ik}^-, c_{ik}^+\right]\right| + \rho \max\limits_i \max\limits_k \left|\left[u_0^-(k), u_0^+(k)\right] - \left[c_{ik}^-, c_{ik}^+\right]\right|}{\left|\left[u_0^-(k), u_0^+(k)\right] - \left[c_{ik}^-, c_{ik}^+\right]\right| + \rho \max\limits_i \max\limits_k \left|\left[u_0^-(k), u_0^+(k)\right] - \left[c_{ik}^-, c_{ik}^+\right]\right|}.$$

The resolving coefficient $\rho \in (0, +\infty)$ is used. The smaller ρ, the greater its resolving power. Usually, $\rho \in [0, 1]$. The value of ρ reflects the degree to which the minimum scores are emphasized relative to the maximum scores. A value of 1.0 would give equal weighting.

After calculating $\xi_i(k)$, the connection between i^{th} alternative and reference number sequence will be calculated according to the following formula

$$r_i = \frac{1}{n} \cdot \sum_{k=1}^{n} \xi_i(k), \qquad i = 1, 2, \ldots, m.$$

STEP 7: DETERMINE OPTIMAL ALTERNATIVE.

The feasible alternative X_t is optimal by grey related analysis if $r_t = \max_{1 \le i \le m} r_i$.

3. EXAMPLE

Here, we shall analyse the following example with the method of grey related analysis to multiple attribute decision making problem with interval numbers. Assume a multiple attribute decision making problem to select a form of enterprise planning system with interval numbers has four feasible alternatives X_1 (application service provider), X_2 (vendor system), X_3 (customize vendor system), and X_4 (best-of-breed system), with five attributes: G_1 (reliability and adaptability of the system), G_2 (flexibility), G_3 (control), G_4 (installation cost), and G_5 (methods improvement). The interval number decision matrix A contains decision maker estimates of alternative performances on different scales as follows.

Table 1. Step 2 : Transform "contrary index" into positive index.

	G_1 Reliable	G_2 Flexible	G_3 Control	G_4 Installation	G_5 Methods
X_1 ASP	[0.5,1.5]	[7.0,9.0]	[5.0,7.0]	[8.5,9.5]	[6.5,7.5]
X_2 Vendor	[2.5,3.5]	[7.5,8.5]	[6.0,8.0]	[6.5,7.5]	[4.5,5.5]
X_3 Customize	[2.5,3.5]	[3.0,5.0]	[8.5,9.5]	[7.5,8.5]	[10.5,11.5]
X_4 Best-of-Breed	[1.0,3.0]	[5.5,6.5]	[9.5,10.5]	[5.5,6.5]	[8.5,9.5]

The weights w_1, w_2, w_3, w_4, w_5, of attributes G_1, G_2, G_3, G_4, G_5 are uncertain, but the experts can specify the following weight ranges: $w_1 \in [0.00, 0.10]$, $w_2 \in [0.20, 0.25]$, $w_3 \in [0.10, 0.15]$, $w_4 \in [0.25, 0.30]$, $w_5 \in [0.30, 0.35]$.

Without loss of generality, we suppose that all the index values are positive.

(1) Standardize the interval number decision matrix A.

Let A_1, A_2, A_3, A_4, A_5 denote the close interval column vector of index interval number decision matrix A, respectively, then $\|A_1\| = 3.5$, $\|A_2\| = 9$, $\|A_3\| = 10.5$, $\|A_4\| = 9.5$, $\|A_5\| = 11.5$.

Standardizing the interval number decision matrix converts the initial divergent measures to a common 0-1 scale. Here, we obtain matrix R as follows.

$$R = \begin{bmatrix} [0.1429, 0.4286] & [0.7778, 1.0000] & [0.4762, 0.6667] & [0.8947, 1.000] & [0.5652, 0.6522] \\ [0.7143, 1.0000] & [0.8333, 0.9444] & [0.5714, 0.7619] & [0.6842, 0.7895] & [0.3913, 0.4783] \\ [0.7143, 1.0000] & [0.3333, 0.5556] & [0.8095, 0.9048] & [0.7895, 0.8947] & [0.9130, 1.0000] \\ [0.2857, 0.8571] & [0.6111, 0.7222] & [0.9048, 1.0000] & [0.5789, 0.6842] & [0.7391, 0.8261] \end{bmatrix}$$

(2) Calculate the interval number weighted decision matrix C by multiplying the weight intervals by matrix R.

$$C = \begin{bmatrix} [0.0000, 0.0429] & [0.1556, 0.2500] & [0.0476, 0.1000] & [0.2237, 0.3000] & [0.1696, 0.2283] \\ [0.0000, 0.1000] & [0.1667, 0.2361] & [0.0571, 0.1143] & [0.1711, 0.2369] & [0.1174, 0.1674] \\ [0.0000, 0.1000] & [0.0667, 0.1389] & [0.0810, 0.1357] & [0.1974, 0.2684] & [0.2739, 0.3500] \\ [0.0000, 0.0857] & [0.1222, 0.1806] & [0.0905, 0.1500] & [0.1447, 0.2053] & [0.2217, 0.2891] \end{bmatrix}$$

(3) Determine the reference number sequence U_0.

$$U_0 = (\ [0, 0.1] \quad [0.1667, 0.25] \quad [0.0905, 0.15] \quad [0.2237, 0.30] \quad [0.2739, 0.35]\ )$$

(4) Calculate the connection between the sequence composed of weighted interval number standardizing index value of every alternative and reference number sequence.

Table 2.

	G_1	G_2	G_3	G_4	G_5	$\min\limits_{k} \Delta_i(k)$	$\max\limits_{k} \Delta_i(k)$
$\Delta_1(k)$	0.0571	0.0111	0.0500	0.0000	0.1217	0	0.1217
$\Delta_2(k)$	0.0000	0.0139	0.0357	0.0631	0.1826	0	0.1826
$\Delta_3(k)$	0.0000	0.1111	0.0143	0.0316	0.0000	0	0.1111
$\Delta_4(k)$	0.0143	0.0694	0.0000	0.0947	0.0609	0	0.0947
$\min\limits_{i}\min\limits_{k} \Delta_i(k)$						0	
$\max\limits_{i}\max\limits_{k} \Delta_i(k)$							0.1826

Let $\Delta_i(k) = |[u_0^-(k), u_0^+(k)] - [c_{ik}^-, c_{ik}^+]|$. The connection coefficient $\xi_i(k)$ (a distance function) is then calculated by the formula as follows,

$$\xi_i(k) = \frac{\min\limits_{i}\min\limits_{k} \Delta_i(k) + \rho \max\limits_{i}\max\limits_{k} \Delta_i(k)}{\Delta_i(k) + \rho \max\limits_{i}\max\limits_{k} \Delta_i(k)}.$$

In the example, $\rho = 0.5$. When $\xi_i(k)$ is determined $\min_i \min_k \Delta_i(k)$ and $\max_i$, $\max_k$, $\Delta_i(k)$ will be calculated as follows.

From the above chart (Table 2), we know that $\min_i \min_k \Delta_i(k) = 0$, $\max_i \max_k \Delta_i(k) = 0.1826$. This is used in the connection coefficient formula to identify distances (larger values means greater distance).

$$\xi(1) = (0.6151, 8915, 0.6462, 1.0000, 0.4286)$$
$$\xi(2) = (1.0000, 0.8680, 0.7188, 0.5911, 0.3333)$$
$$\xi(3) = (1.0000, 0.4511, 0.8647, 0.7430, 1.0000)$$
$$\xi(4) = (0.8647, 0.5680, 1.0000, 0.4908, 0.6000)$$

By these results, we know that the connection between every alternative and reference number sequence is, respectively $r_1 = 0.7163$, $r_2 = 0.7022$, $r_3 = 0.8118$, $r_4 = 0.7047$.

(5) Ranking feasible alternatives from largest to smallest r_i the rank order of feasible alternatives is $X_3 > X_1 > X_4 > X_2$.

Here the rank ordering implies that for the ranges of weights and alternative measures given, the alternative to customize a vendor system is ranked first, followed by the ASP option, the best-of-breed option, and finally the full vendor system. The connection coefficients indicate that the customized vendor system is quite a bit better than the other three alternatives.

4. CONCLUSION

The method of grey related analysis to multiple attribute decision making problem with interval numbers given in this paper concerns interval fuzzy input parameters. It applies the traditional method of grey related analysis.

The method reflects decision maker or group uncertainty concerning multiple criteria decision input parameters. The method presented here is simple, practical, and requires less rigid input from decision makers. Weight inputs are entered as fuzzy interval numbers. Alternative performance scores also can be entered as general interval fuzzy numbers. Both weight and performance scores are standardized, and composite utility value ranges obtained. Connection coefficients are generated that serve as the basis for identifying the distance of each alternative from the nadir solution.

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