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Implementation of the centroid method of Solymosi and Dombi

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Abstract: Multiple objective decision support, either for selection among finite alternatives or for combining multiple functions in mathematical programming, often involves estimating a set of weights of relative importance. Recently Solymosi and Dombi proposed a means of estimating such weights based upon ordinal preference information. This method relies upon the centroid of bounded weights. Analytic hierarchy process uses ratio estimates of pairwise comparisons. Using only ordinal information, the centroid of weights is developed as an alternative means of weighting hierarchical elements. These weights could be used in selection among alternatives directly, in concordance analysis, or in multiple objective linear programming. Formulation for the centroid of feasible weight values is presented with a table of values for cases where the preference ranking includes no ties. Comparison of the technique on a student job selection decision indicates slight, but insignificant, decline in accuracy relative to the analytic hierarchy based method.

Keywords: Multiobjective selection, analytic hierarchy process, determination of weights

Introduction

A number of techniques have been developed for obtaining sets of weights for combining multiobjective functions. These approaches include multiattribute utility theory (MAUT) [2,6], the analytic hierarchy process (AHP) [12,13,14], regression [7], and simple weighting and ranking techniques. Each of these techniques requires varying levels of input from decision makers, but the intent of all of them is to provide a means of selecting among a set of alternative potential decisions X_j {j alternatives} while reflecting multiple decision objectives O_k {k objectives}. The assumption is generally made that for each alternative X_j, a measure of value V_{jk} can be obtained (objectively or subjectively) for each objective O_k. With the exception of MAUT (which can adopt a nonlinear estimate of utility), these methods share a resulting additive value function of the form:

$$\text{Value}(X_j) = \sum_{i=1}^{k} W_k V_{jk} X_j \quad \text{for } j = 1, \ldots, J.$$

Usually, $\sum_{i=1}^{k} W_k = 1$, which can easily be accomplished by normalization. The weights, W_k, can be viewed as the relative importance of each objective k. Note that the measures of value V_{jk} should be scaled to a common metric, such as a maximum of 1 for an ideal measure, and a minimum of 0 for a totally unacceptable measure, in order not to dilute the relative importance provided by the weights W_k. Belton [2] discussed the differences in AHP and MAUT (multi-attribute value function in Belton's terminology) for eliciting weights.

Potential uses of the resulting set of weights W_k include direct assessment of alternatives, as an input to concordance analysis [11], in consideration of risk [5], or as an initial set of weights in multiple objective linear programming (MOLP) [19,20]. At least three studies utilizing AHP in MOLP have been published [1,9,10]. Usually, use of the weights in multiobjective analysis are as a means for filtering a long list of alternatives down to a shorter list for more detailed consideration by decision makers. Development of a set of factors important for a decision along with a set of weights could also be used as an objective means of selection, such as in employment.

Each of the techniques to obtain W_k have varying amounts of decision maker input. MAUT can involve a fairly extensive examination of tradeoffs between objectives. AHP relies upon subjective pairwise comparison of hierarchy elements. A regression approach would require a subjective assessment of overall value for each sample alternative X_j. The weighting and rating approach would require less input, as decision makers would simply subjectively assign the W_k. In general, one would expect that the more effort that was devoted to the approximation of W_k, the more accurate the resulting weights.

Dyer [4] recently argued that elicitation questions in AHP aim at determining strength of preference, and thus require subjective estimates on a cardinal scale. Dyer noted that this strength of preference approach has been criticized in the literature. Saaty [16] responded that AHP has always been understood to be a ratio technique, which can be used to obtain relative measures when absolute measures are not available. AHP, through pairwise comparisons of hierarchical factors, is often used to convert subjective estimates of relative importance into weights. Those articles discuss other points not germane here. But that debate identifies an issue concerning the means of eliciting preference information.

Recently, Solymosi and Dombi [18] have presented a technique which would require decision makers to determine the k objective factors, and simply rank the relative importance of each of these factors. Their technique essentially seeks to take the implicit relationship of the k weights, and find the centroid of the area bounded by these relationships. If the hierarchy of AHP is used to identify measurable factors of importance in a decision context, ordinal preference information can be used instead of the ratio method of pairwise comparisons. Since the ratio information, when applied to subjective measures, is approximate anyway, the centroid of bounded weights provides a means to convert more robust estimates of weights at much less effort in preference elicitation.

The purpose of this paper is to provide a technique for obtaining the weights for Solymosi and Dombi's (S&D) technique more rapidly. This technique is attractive because the hierarchical structure of AHP can be combined with less input from the decision maker. It is expected that the resulting set of weights will be less precise, but the degree of relative imprecision is expected to be minimal. In the same sense as regression, error will be minimized by finding the centroid of weight limits. The paper will also present experimental results comparing AHP and S&D based approaches on 46 students, faced with a job selection decision. The purpose of this experiment was to determine the relative accuracy of both techniques for a decision environment where subjects understood the problem well, and were faced with a decision important to them. The decision proposed was to select a position upon graduation. Because the subject students were graduating within one year, and were in the process of employment interviewing, the decision is purported to be a valid one.

Analytic Hierarchy Process

AHP provides a means to convert subjective assessments into a scalar relative value. Three steps are involved: (1) problem decomposition; (2) comparative judgement; and (3) synthesis. Problems are decomposed, yielding a hierarchy of objective factors. The intent is for the decision maker to develop a collectively exhaustive list of objective factors bearing upon a decision. Because of limitations of concentration, these factors are arranged in a hierarchy of elements and subelements. Saaty [12] recommends no more than seven subelements for consideration at one time. Subjective assessment is accomplished by subjective pairwise comparison at each node of the hierarchy. Saaty recommends use of the eigen vector of each pairwise comparison matrix in order to gain a consistent estimate of relative weights (see Belton [2] for discussion). Krovak [7] compared AHP with three regression based techniques for obtaining weight estimates. Krovak found that for consistent matrices, all four methods yielded similar results, although regression based approaches might be more appropriate in circumstances of high inconsistency. Use of regression based approaches would involve more complex analysis. Saaty would apply all alternative decisions as the bottom level of the hierarchy, continuing the subjective comparison of each alternative with respect to each objective factor.

The hierarchy of objectives obtained by AHP could also be used to obtain a scalar set of weights, which could then be applied to a multitude of decision alternatives. While this approach loses some of the 'purity' of AHP (Saaty [15]), the technique is attractive because it provides a means to identify the criteria of importance, and a set of weights for these criteria can be obtained which can then be applied to a large number of alternatives. A limitation of AHP is that the number of pairwise comparisons required can be substantial. Further, the 'pure' AHP approach would only allow consideration of a maximum of seven alternatives at one time.

To demonstrate how AHP would work, consider the decision facing students nearing graduation. Hopefully, there would be employment alternatives available to them. A number of factors of importance might bear upon each student's decision.

The first step in AHP is to identify those employment features important to the decision maker. Such a list might consider the pay associated with a job offer, but would likely include other factors as well. Such factors might include job location, type of work, opportunity for advancement, and working conditions. Furthermore, each of these overall criteria may include subelements. An example of subelements for pay might include starting pay, which could consist of a precisely measurable salary, but it could also consist of commission opportunities (involving some subjective estimation), raise opportunities, insurance package, etc. Once factors have been identified, the analytic hierarchy process would involve organizing these factors into a hierarchical structure, seeking no more than seven elements at any one node of a tree (cf. Figure 1). Each of the primary factors could be subdivided as Pay was in this example. In turn, each subelement could be subdivided. Altogether, it is assumed that the hierarchy is collectively exhaustive, in that every factor of importance will be included.

The second phase of AHP is to conduct pairwise comparisons at each node of this hierarchy. For instance, the overall relative importance of primary factors would be obtained through ten pairwise comparisons (between Pay and the other four factors, Location and the remaining three factors, and so on). Further, every factor that is subdivided would also require a pairwise comparison matrix. In AHP, these pairwise comparisons are subjective, asking the decision maker to rank relative importance on a 1 to 9 scale, where 1 indicates equal importance, 9 represents overwhelming importance of the base factor to the other factor, and all numbers in between representing relative degrees of importance. The matrix of pairwise comparisons is square, with a

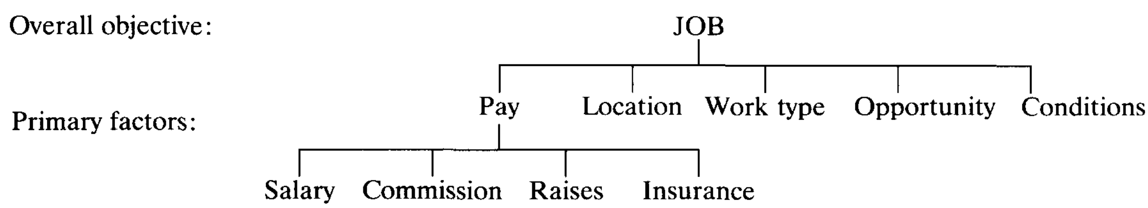

Figure 1

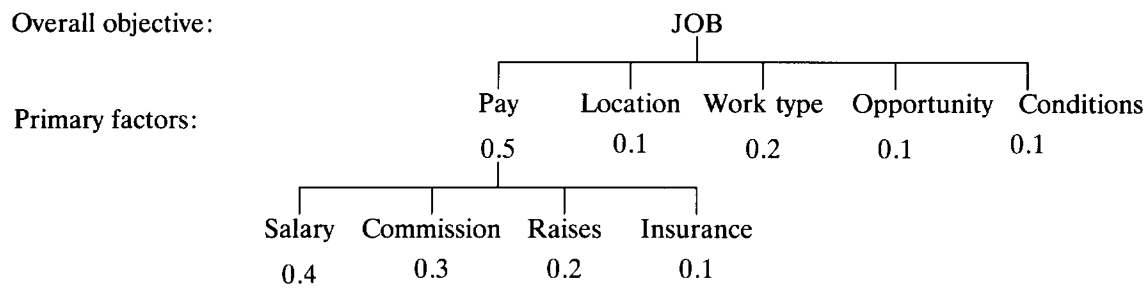

Figure 2

diagonal of 1 (all factors are equal in importance to themselves), and symmetric elements are inverse (if Pay has a relative importance to Location of 5, the entry for Pay in the Location row is $\frac{1}{5}$).

This information contained in the matrices can then be mathematically analyzed by determining the eigen vector for each pairwise comparison matrix. The normalized eigen vector would be the relative weights for each factor (cf. Figure 2). In this example, one pairwise comparison matrix would yield a normalized set of weights for the primary factors. Pay's subordinate elements would have a pairwise comparison matrix of four elements, also yielding a set of normalized weights. AHP would involve further pairwise comparisons listing each job opportunity below each hierarchical element which did not have subordinate elements. Synthesis is obtained by the weights of each superior element being divided between all subordinate elements.

A set of weights for all elements which did not have subordinate elements could also be inferred. For instance, the 0.5 weight on Pay elements would be divided proportionately between the four subordinate factors of Salary, Commission, Raises, and Insurance. In this partial example, for instance, the overall formula would be:

$$\begin{aligned} &0.2 \text{ Salary} + 0.15 \text{ Commission} + 0.1 \text{ Raises} \\ &\quad + 0.05 \text{ Insurance} + 0.1 \text{ Location} \\ &\quad + 0.2 \text{ Work type} + 0.1 \text{ Opportunity} \\ &\quad + 0.1 \text{ Conditions}. \end{aligned}$$

Of course, factors such as Location (a primary factor) or Raises (a subelement) could be further subdivided.

While AHP was intended to avoid the inaccuracy inherent in transporting such a formula to alternatives being considered by using the alternatives themselves in the hierarchy, it is possible to utilize the hierarchy to develop a set of weights used for a variety of purposes. Many multiobjective techniques assume decision makers have these relative weights of importance (ELECTRE [11]), or use the resulting linear estimate of utility as a basis for further analysis (multiobjective programming, such as the Method of Zionts and Wallenius [20], Steuer's Method [19]). And there are many contexts where transportation of the set of weights is useful, as in decision making rules designed to be as fair as possible through elimination of subjective scoring of applicants for a position.

In order to transport the weights to another decison, a scoring system for each formula factor is necessary. It is important that no distortion of scale be introduced, because the analysis yielding the relative importance of each factor has been conducted. Therefore, a rating for each decision alternative on each factor (salary, commission, work type, etc.), must be developed by the decision maker. Belton [2] noted that many decision makers find it useful to evaluate the ideal characteristic as 100, and the worst possible characteristic as 0. This approach allows decision makers to reflect nonlinear utility, in that the 100 points can be allocated in any manner reflecting decision maker intent.

The centroid method

Solymosi and Dombi [18] presented a technique which would yield a set of weights for multiple objectives based upon preference information among pairs of criteria. This approach was intended to be interactive, in that decision makers would make as many preference selections as were necessary to yield acceptable weights. The essence of the technique is that preference information between criteria yields knowledge about the bounds of specific weight values. Individual weights could take on a range of values. Solymosi and Dombi used the centroid of the bounded area as a likely estimate of the true weights implied by preference statements. The basis for this approach is to minimize the maximum error by finding the weights in the center of the region bounded by the decision maker's ordinal ranking of factors.

Once a hierarchy of factors is obtained from phase 1 of AHP, preference information can be obtained as an alternative to the pairwise comparisons. In fact, all decision makers would have to do would be to rank order (with preference or equality) all factors in the hierarchy which did not have subordinate elements. For instance, in the example used above, a possible preference ranking might be:

Salary = Work type > Commission > Raises
= Location = Opportunity
= Conditions > Insurance

is implied by the simple weights used in the example. Note that pairwise comparisons would be unlikely to yield so many ties (even if there really were indifference). An expected advantage of the centroid approach is that all factors are considered directly. In AHP, a potential problem is that subelements of one branch of the hierarchy are never directly compared to elements in the other branches. As in AHP, the sum of the weights in the centroid approach for these eight factors would add to one (be normalized).

$$W_s + W_w + W_{com} + W_r + W_\ell + W_o + W_{con} + W_i = 1. \tag{1}$$

The preference information would yield other relationships:

$$\begin{aligned} & W_s = W_w, \\ & W_w > W_{com}, && (\text{at the limit, } W_w \geq W_{com}) \\ & W_{com} > W_r, && (\text{at the limit, } W_{com} \geq W_r) \\ & W_r = W_\ell = W_o = W_{con}, \\ & W_{con} > W_i, && (\text{at the limit, } W_{con} \geq W_i). \end{aligned} \tag{2}$$

Table 1

W_s	W_w	W_{com}	W_r	W_ℓ	W_o	W_{con}	W_i
0.5	0.5	0	0	0	0	0	0
0.333	0.333	0.333	0	0	0	0	0
0.143	0.143	0.143	0.143	0.143	0.143	0.143	0
0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125
0.2753	0.2753	0.1503	0.0670	0.0670	0.0670	0.0670	0.0312

There are many possible sets of weights satisfying these conditions simultaneously. Examples are shown in Table 1 (considering the limit of > is ≥). In fact, these are the extreme points of the bounded set of weights. An infinite number of interior points to the bounds provided by the set of constraints (1) and (2) exist. W_s could range from 0.125 to 0.5. W_i could range from 0 to 0.125. Solymosi and Dombi suggest the centroid, obtained by averaging all extreme points, as an estimate of the true set of weights. In this case, this approach would yield (using accurate fractions) the last row of Table 1. This approach is attractive, in that it combines the structured means of identifying objective factors provided with AHP, with a straightforward means of obtaining factor weights by utilizing preference information from the decision maker. Solymosi and Dombi proposed an iterative procedure which would elicit preference information until the decision maker was satisfied with the resulting weights. However, by using the AHP approach, more control over collectively exhausting all factors of importance is obtained. Once that is done, simple preference ordering of all factors provides information which can be used to infer weights for each factor in a manner such less involved than the AHP technique. Of course, the AHP technique would be expected to yield more precise weights, but at the cost of potentially many pairwise comparisons.

Note that if all relationships are strict preferences, the set of weights can be determined by formula. For n factors, the weight of each factor will be for factor $k = 1$ to n, $\{\sum_{i=k}^{n}(1/i)\}/n$. A table of values for n factors is appears in the appendix.

Experiment

As part of their coursework, students were assigned a problem applying AHP to the decision of selecting among alternative employment opportunities. The experiment was designed in three phases: Phase one was for the students to develop a hierarchy of job factors important to them. To facilitate this phase, eight typical jobs, with job description, type of employer, location, starting pay, promotion potential, job risk, and working schedule was provided. The precise assignment is given in Figure 3. The intent was for the students to develop a formula, which could then be tested upon another set of jobs in a later phase. This was done by each student developing their own set of factors, based upon the factors considered important to them personally. These factors were arranged in an appropriate hierarchy (avoiding more than seven factors branching from any one node), and pairwise comparisons conducted. The eigen vector for each pairwise com-

Place yourself in the position of being in the job market. Identify factors and subfactors you feel are important in your personal opinion. Real stuff. The only requirement is that you list factors in more than one level. Assume you have the following opportunities. You are welcome to ask for further details regarding characteristics.

A. Database technician, major petroleum company – Houston, TX,
$30,000 per year, high promotion potential,
high risk of job loss,
8–5 5 days per week, 50 weeks per year.

B. Computer consultant – on campus – local community,
$18,000 per year, slow promotion potential,
low risk of job loss,
8–5, 5 days per week, 48 weeks per year.

C. Management trainee – EDS – Flint, MI,
$24,000 per year, fast promotion potential,
high risk of job loss,
7–6, 5 days + saturdays, 50 weeks per year.

D. Beginning information systems analyst - major firm - Dallas, TX,
$30,000 per year, moderate promotion potential,
moderate risk of job loss,
8–5, 5 days per week, 50 weeks per year, no overtime.

E. Information systems analyst - small firm - St. Louis, MO,
$28,000 per year, moderate promotion potential,
moderate risk of job loss,
7–6, 5 days per week, 50 weeks per year, lots of unpaid overtime.

F. Software development - small firm - Phoenix, AZ,
$28,000 per year, moderate promotion potential,
moderate risk of job loss,
9–6, 5 days per week, 50 weeks per year, some unpaid overtime.

G. Maintenance programming - Oil company - Houston, TX
$27,000 per year, slow promotion potential,
moderate risk of job loss,
8–5, 5 days per week, 50 weeks per year, all overtime paid.

H. Independent (self-employed) computer consultant - Houston, TX,
unknown pay, but pay MAY average $100/hour,
should average 70 hours/month, 10 months/year,
might only be 40 hours/month, could be 120 hour/month.

This exercise is to develop a formula with AHP. You do not need to turn it in at this time (wait until you get the test batch of jobs).

1. Develop your AHP formula.
2 For those factors in the AHP formula, rank order the factors by importance. (When rank ordering, > indicates preference, = indicates indifference)

Figure 3. Original alternatives

parison was obtained, providing the set of weights. The eigen vector is a function of the maximum eigen value of the pairwise comparison matrix. An inconsistency index is also available to provide a check on the consistency of the multiple ratings in the pairwise comparison matrix. All inconsistency indices in the study were less than the 0.1 limit proposed by Saaty, as students revised their pairwise comparisons if a larger inconsistency index was encountered.

Phase two of the assignment was to apply the formula to another set of job opportunities, with the same described elements as in phase one, but with only seven jobs at this stage (Figure 4). Note that it is expected that some of the accuracy of AHP as developed by Saaty is lost. However, the intent was to determine the relative accuracy of a set of weights developed by AHP and by S&D. Students were also requested to simply provide the ordinal rank of each of their factors. This was

You have developed your formula for job selection. Please test your formula on the following jobs:

A. Data processing specialist – bank in College Station,
$16,000 per year, slow promotion potential,
low risk of job loss,
8–5, 5 days per week, 50 weeks per year.

B. Computer consultant – on campus – local community,
$18,000 per year, slow promotion potential,
low risk of job loss,
8–5, 5 days per week, 48 weeks per year.

C. Freelance computer consultant – Brazos County,
unknown pay – but business MAY average $100/hour,
50 hours/month – probably 11 months/year,
could be 10 hours/month, could be 100 hours/month.

D. Beginning information systems analyst – Big 8 firm – Dallas, TX,
$31,000 per year, moderate promotion potential,
high risk of job loss,
7–6, 5 days per week 50 weeks per year, lots of paid overtime.

E. Software sales – small international firm – Houston, TX,
$27,000 per year, high promotion potential,
moderate risk of job loss,
7–6, 5 days per week, 50 weeks per year, lots of unpaid overtime.

F. Beginning information systems analyst, major firm – Fresno, CA,
$32,000 per year, moderate promotion potential,
moderate risk of job loss,
9–6, 5 days per week, 50 weeks per year, all overtime paid.

G. Maintenance programming – oil company – New Orleans, LA,
$29,000 per year, slow promotion potential,
moderate risk of job loss,
8–5, 5 days per week, 50 weeks per year, all overtime paid.

1. Score these jobs according to the scale you developed for the assignment and apply the formula developed in the assignment to these jobs.
2. USING YOUR JUDGEMENT, list your preference for these seven jobs.
3. For those factors in YOUR AHP formula, rank order the FACTORS by importance. (What I am going to do is try an alternative method. I need your JUDGEMENTAL impression of factor importance.)

For instance, if your formula is:

0.45 PAY + 0.20 POTENTIAL + 0.15 LOCATION + 0.10 RISK + 0.10 TYPE OF WORK

I want you to say what order of importance these five factors should have in your opinion. That might be: PAY > POTENTIAL > TYPE OF WORK > LOCATION > RISK Ties are = . The list may be or may not be the same as the formula implies.

Figure 4. Test set of alternatives

Factors	AHP rank	Centroid rank	Post analysis (holistic)	Hotelling–Pabst (Σ diff²)			First choice (Σ diff²)	
				AHP-post	Cent-post	AHP-cent	AHP/post	Cent/post
3	F>D>C>E>G>A=B	E>D>F>C>G>A=B	F>E>D>C>G>A>B	6.5 ***	6.5 ***	14 **	1	0
4	D>E>F>G>C>A>B	D>E>F>G>C>A>B	D>E>F>G>C>A>B	0 ***	0 ***	0 ***	1	1
4	E>D>C>A>G>B>F	E>D>C>G>A>B>F	G>E>F>D>B>A>C	58	48	2 ***	0	0
5	D>F>G>E>B>A>C	D>E>F>G>C>B>A	D>G>E>F>B>A>C	6 ***	12 **	12 **	1	1
5	F>G>D>C>E>B>A	F>F>C>D>E>B>A	F>G>E>D>C>B>A	6 ***	8 ***	2 ***	1	1
6	B>F>G>D>E>A>C	D>F>G>E>C>B>A	F>D>G>E>B>A>C	22 *	8 ***	40	0	0
6	D>C>E>B>F>A>G	D>C>B>A>E>F>G	D>E>C>B>F>G>A	4 ***	22 *	10 **	1	1
6	D>E>A>C>B>F>G	D>E>C>A>B>F>G	F>D>G>E>B>A>C	64	66	2 ***	0	0
6	D>F>C>B>E>A>G	D>F>C>B>E>A>G	D>F>C>B>E>A>G	0 ***	0 ***	0 ***	1	1
6	D>F>E>G>C>A=B	D>F>E>G>C>A=B	D>F>E>G>C>B>A	0.5 ***	0.5 ***	0 ***	1	1
6	F>C>D>B>E>G>A	F>C>D>E>B>G>A	F>D>G>E>C>B>A	24 *	20 *	2 ***	1	1
6	F>C>D>E>G>B>A	F>D>C>E>G>B>A	F>D>E>C>G>B>A	6 ***	2 ***	2 ***	1	1
6	F>C>D>G>E>B>A	C>F>D>G>E>B>A	F>D>G>E>C>B>A	12 **	20 *	2 ***	1	0
6	F>D>E>C>G>B>A	F>D>E>G>C>B>A	D>G>F>E>C>B>A	16 **	10 **	2 ***	0	0
6	F>D>G>E>B>C>A	D>F>G>E>tB>C>A	D>F>G>E>B>C>A	2 ***	0 ***	2 ***	0	1
7	C>E>F>D>B>A>G	C>E>F>D>B>A>G	C>E>F>D>B>A>G	0 ***	0 ***	0 ***	1	1
7	D>F>E>G>A>B>C	F>D>E>G>B>A>C	E>F>D>G>C>B>A	16 **	12 **	4 ***	0	0
7	D>F>G>C>E>A>B	D>F>G>E>C>A>B	D>F>C>E>G>B>A	8 **	10 **	2 ***	1	1
7	E>D>B>G>C>A>F	E>D>B>G>F>C>A	E>D>G>B>F>C>A	8 ***	2 ***	6 ***	1	1
7	E>G>D>A>B>C>F	E>D>A>B>C>G>F	E>D>G=F>C>B>A	26.5	38.5	20 *	1	1
7	F>C>D>G>E>A=B	F>D>C>G>E>A=B	F>C>D>G>E>A=B	0 ***	2 ***	2 ***	1	1
7	F>D>E>G>B=C>A	F>D>G>E>B=C>A	F>G>D>E>B>C>A	6.5 ***	2.5 ***	2 ***	1	1
8	C>F>E>D>B>A>G	C>E>F>D>B>A>G	E>C>D>F>G>B>A	16 **	10 **	2 ***	0	0
8	D>C>F>E>G>B>A	D>C>F>E>G>B>A	D>C>F>E>G>B>A	0 ***	0 ***	0 ***	1	1
8	D>F>E>G>B>A>C	D>E>F>G>B>A>C	D>E>F>G>C>B>A	8 ***	6 ***	2 ***	1	1
8	D>F>G>E>C>A>B	F>D>E>C>G>A>B	D>E>G>F>A>B>C	14 **	25 *	8 ***	1	0

8	E>D>C>B>A>F>G	E>D>F>C>B>A>G	E>D>G>B>A>C>	26 *	38	12 **	1	1
8	F>D>C>G>B>E>A	F>D>C>B>G>E>A	F>D>G>E>C>B>A	10 **	16 **	2 ***	1	1
8	F>D>E>C>G>A>B	D>F>G>E>C>A>B	D>F>G>A>B>C>E	34	18 **	8 ***	0	1
8	F>D>E>G>A>B>C	F>D>E>G>A>B>C	D>F>E>G>A>B>C	2 ***	2 ***	0 ***	0	0
8	F>D>G>C>E>B>A	F>D>G>E>C>B>A	D>F>E>G>C>B>A	8 ***	4 ***	2 ***	0	0
8	F>G>C>B>D>E>A	F>G>C>B>D>E>A	F>G>C>D>E>B>A	6 ***	6 ***	0 ***	1	1
9	C>D>F>E>G>A>B	C>F>D>E>G>B>A	C>F>D>G>E>B>A	6 ***	2 ***	4 ***	1	1
9	D>F>C>E>G>B>A	D>F>E>C>G>B>A	D>F>C>G>E>B>A	2 ***	6 ***	2 ***	1	1
9	D>G>F>C>B>E>A	D>C>B>G>F>A>E	D>F>G>E>C>B>A	8 ***	38	18 **	1	1
9	E>D>G>F>B>A>C	E>G>D>F>B>A>C	E>D>G>F>B>A>C	0 ***	2 ***	2 ***	1	1
9	F>D>E>G>B>A>C	F>D>E>G>B>A>C	F>D>E>G>B>A>C	0 ***	0 ***	0 ***	1	1
9	F>D>G>C>E>B>A	F>D>G>E>C>A>B	F>D>C>G>B>E>A	4 ***	14 **	4 ***	1	1
9	G>F>D>C>E>A>B	G>D>F>A>C>B>E	G>D>E>F>B>A>C	22 *	26 *	12 **	1	1
10	D>F>B>C>E>G>A	D>F>E>B>C>G>A	D>G>F>E>C>B>A	28	22 *	6 ***	1	1
10	F>D>G>E>A>B>C	D>F>E>G>A>B>C	E=G>F>D>C>B>A	24.5 *	26.5	4 ***	0	0
10	F>D>G>E>C>A>B	F>D>G>E>C>A>B	F>D>G>E>C>B>A	2 ***	2 ***	0 ***	1	1
11	D>F>G>E>A=B>C	D>F>G>E>A=B>C	D>F>G>E>A=B>C	0 ***	0 ***	0 ***	1	1
11	F>D>C>G>E>B>A	F>D>B>C>G>E>A	F>D>C>G>E>B>A	0 ***	12 **	12 **	1	1
12	E>C>D>A>B>F>G	F>D>E>G>C>B>A	E>D>C>A>B>F>G	2 ***	52	58	1	0
14	F>E>C>G>D>B>A	F>D>E>G>C>B>A	E>G>D>C>F>A>B	28	28	14 **	0	0
46 observations	0.99 significance ***			29 (0.630)	23 (0.500)	35 (0.761)		
	0.95 significance **			6 (0.130)	9(0.196)	8 (0.174)		
	0.90 significance *			5 (0.109)	6 (0.130)	1 (0.022)		
	not significant			6 (0.130)	8 (0.174)	2 (0.043)		

figures in parentheses are observations/46

AHP/POST same first choice 34/46 = 0.739

CENT/POST same first choice 32/46 = 0.696

AHP/CENT same first choice 37/46 = 0.804

Figure 5. Comparisons of results by method

the only student input to the centroid method. The hierarchy was used as the source of the factors. This preference information was then used by the instructor to determine weights using the S&D method.

Phase three of the assignment was for the students to rank each job. The reason for making this comparison at the end of the exercise was to obtain as accurate a base judgement as possible, without an exhaustive analysis. Schoemaker and Waid [17] use the holistic assessment of decision makers as the base for comparison. Fischer compared MUT and holistic procedures for convergence, concluding that convergence will occur with iteration [5]. Lai and Hopkins [8] have noted the difficulty of estimating the 'correct' outcome of such a decision. While the judgement of the decision maker would be expected to be 'correct', one of the benefits of the AHP process is to educate the decision maker about the factors involved in the decision. Hopkins suggests an iterative process, using multiple methods, converging upon the 'correct' decision. In this experiment, it is realized that the basis for comparison is imperfect, but subject judgement should be a sound basis.

Results

Figure 5 presents the results of the experiment. Three rank orders were obtained. The first rank order was the AHP ranking determining by each student, applying student scoring of each objective factor on each job. This was done so that the AHP derived formula could be transported to another decision. The second rank order (centroid) was determined by the instructor, who took student preference order of objective factors and applied student scores for factors by job. The third rank order (post) was the post analysis student preference ranking of the seven job opportunities. The Hotelling–Pabst statistic was used to compare the resulting rank orders.

The Hotelling–Pabst statistic (H–P) is:

$$\sum_{i=1}^{n} [R(X) - R(Y)]^2$$

where $R(X)$ is the rank order of one method, and $R(Y)$ is the rank order of the other method. In effect, the statistic is the sum of squared differences in ranks. The significance levels for seven items is: H–P ≤ 8 significant at 0.99 [3]

H–P ≤ 18 significant at 0.95,

H–P ≤ 26 significant at 0.90.

Spearman's ρ is a direct mapping of H–P.

$$\rho = 1 - 6(\text{H–P})/n(n^2 - 1)$$

Spearman's ρ has the benefit of transforming H–P to a 0–1 scale. The H–P score, transformed by the formula for ρ, will have the same confidence limits as Spearman's ρ. H–P was used here for simplicity.

The last two columns of Figure 3 denote whether the test techniques matched the first choice of the post study judgement. A 1 indicates the same choice for the test technique and the post study judgement. A 0 indicates a different first choice. The AHP based approach matched the post study judgement 34 of 46 times, while the centroid approach matched post study judgement 32 or 46 times. The two test techniques resulted in the same first choice in 37 of 46 observations.

AHP-post

The AHP based method was significantly similar to the post ranking in 87% of the samples at the 0.90 level. This seems to be quite good, as the formula was transported (developed on one set of alternatives, and applied to another). Reasons for difference are expected to include inevitable omission of factors considered, because an important decision (such as selection of employment) would include a number of subtle differences which are difficult to express (or even recognize) by decision makers, but which can influence an overall holistic judgement. Further, AHP analysis as presented by Saaty allows some ability to reflect nonlinear benefits. The scoring system used in this study also allows some ability to reflect nonlinear benefit, through allocation of the 0–1 scoring for each job performance rating on each factor. There were 6 cases of the 46 where high differences were found. A number of sources of difference are possible, including student error, as well as the previously mentioned element of incomplete identification of objectives. However, it is contended that most decision makers would not be familiar with the techniques presented to them, and the decision did have salience for the subjects.

Centroid-post

The centroid method involved less student judgement, as their input was the hierarchy developed by AHP, the scoring of each job on each factor (also used in AHP), and a simple preference ranking of factors (allowing ties). Centroid weights were calculated from this information by the instructor, and applied by the instructor to the student scoring system. As would be expected, there was less accuracy in matching the post analysis student rankings, although 82.6% of the 46 subjects had significantly similar rankings at the 0.90 level of confidence.

AHP-centroid

The AHP and centroid methods were very similar in results, with all but two of the subjects having similar results between the two methods. Since precisely the same scoring for job performance by objective was used, this is also expected. Any differences would occur because of the ability of AHP to fine tune weights more precisely. The centroid approach would simply take the middle of the implied feasible region. Note that the number of factors each subject used does not seem to have significant bearing upon accuracy. It would be expected that AHP, with the ability to fine tune weight estimates, would have an advantage over the centroid approach, which uses the centroid of the bounds upon weights. When fewer factors are used, there is a much larger feasible region that would satisfy the restrictions upon weights. Yet review of Figure 3, which is ordered by number of factors used by each subject, shows little pattern in accuracy on the dimension of the number of factors used.

Viewed in terms of matching the decision (alternative ranked first), the AHP based approach matched the holistic analysis 34 of 46 times (73.9%), while the centroid approach matched the holistic analysis 32 of 46 times (69.6%). This would indicate some obvious value of both the AHP and centroid methods in supporting a decision. However, the accuracy is not sufficient to trust with an important decision. This emphasizes the need to apply multiple objective analyses as guides, in the role of decision support, as opposed to preempting decision maker judgement. While the centroid approach was not as accurate in matching the holistic assessment, there were two cases where the AHP approach did not match the holistic first choice, while the centroid technique did.

Conclusions

This study sought to examine the relative accuracy of AHP and of an approach using centroid weights (motivated by the technique of Solymosi and Dombi) in developing sets of weights reflecting the relative importance of multiple objectives. The ability to identify such weights is useful in many decision supporting contexts. One example would be in determining the weights for combining multiple objectives in mathematical programming. Example applications of this were noted. Another use of transportable weights would be in a model designed to identify a subset of attractive alternatives for more detailed decision maker consideration.

The results of the study clearly indicate that neither AHP nor the centroid approach would provide a tool that could be expected to totally reflect decision maker preference. However, both approaches could be relied upon to generally reflect decision maker preferences. This would be useful in sorting through large lists of alternatives, and presenting decision makers with a shortened list of alternatives which should be more attractive.

While the AHP based approach would be expected to be more accurate, the centroid technique has attraction in that it provides nearly as accurate a set of weights, while requiring much less input of a potentially less confusing nature to decision makers. Since most users of decision support packages are not expected to be experts in various techniques, this can be instrumental in the successful delivery of analytic approaches to management. In AHP, decision makers are asked to: 1) identify objectives, as well as subobjectives, and organize them into a hierarchy; and 2) conduct pairwise comparisons at each node of the hierarchy. Step 1) is a useful approach, which allows decision makers to concentrate upon what they want to accomplish. The centroid approach in this study utilized step 1) from the AHP analysis, but substituted a simple preference ranking of factors for the more involved pairwise comparisons. While pairwise comparisons are not difficult to do, the repetitiveness of the operation may be a burden to some decision makers. Nearly as accurate results (when seeking a set of weights) are available from the centroid approach. Other approaches for obtaining weights are also available, but generally require even more involved analysis than the pairwise comparisons of AHP.

Preference information of the factors reflecting multiple objectives can be identified by identifying the extreme points of the bounds upon individual weights. Ordinal ranking of all factors in one step is required of the decision maker. If no ties are present in this preference ranking, a formula for the individual weights as a function of the number of factors was presented. Further, a table of these values was appended. If ties are present, an example calculation was provided.

Appendix

Table A1
Centroid weights for N factors, ties not considered

Weight	N									
	1	2	3	4	5	6	7	8	9	10
1	1.00000	0.75000	0.61111	0.52083	0.45667	0.40833	0.37041	0.33973	0.31433	0.29290
2		0.25000	0.27778	0.27083	0.25667	0.24167	0.22755	0.21473	0.20322	0.19290
3			0.11111	0.14583	0.15667	0.15833	0.15612	0.15223	0.14766	0.14290
4				0.6250	0.9000	0.10278	0.10850	0.11057	0.11063	0.10956
5					0.04000	0.06111	0.07279	0.07932	0.08285	0.08456
6						0.02778	0.04422	0.05432	0.06063	0.06456
7							0.02041	0.03348	0.04211	0.04790
8								0.01563	0.02623	0.03361
9									0.01235	0.02111
10										0.0100

Weight	N									
	11	12	13	14	15	16	17	18	19	20
1	0.27453	0.25860	0.24463	0.23225	0.22122	0.21130	0.20233	0.19417	0.18672	0.17989
2	0.18363	0.17527	0.16770	0.16083	0.15455	0.14880	0.14350	0.13862	0.13409	0.12989
3	0.13817	0.13360	0.12924	0.12511	0.12122	0.11755	0.11409	0.11084	0.10778	0.10489
4	0.10787	0.10582	0.10360	0.10130	0.09899	0.09671	0.09448	0.09232	0.09023	0.08822
5	0.08514	0.08499	0.08437	0.08344	0.08233	0.08109	0.07978	0.07843	0.07707	0.07572
6	0.06696	0.06832	0.06898	0.06916	0.06899	0.06859	0.06801	0.06732	0.06655	0.06572
7	0.05181	0.05443	0.05616	0.05725	0.05788	0.05817	0.05821	0.05806	0.05778	0.05739
8	0.03882	0.04253	0.04518	0.04705	0.04836	0.04924	0.04981	0.05013	0.05026	0.05024
9	0.02746	0.03211	0.03556	0.03812	0.04002	0.04143	0.04245	0.04318	0.04368	0.04399
10	0.01736	0.02285	0.02701	0.03019	0.03262	0.03449	0.03592	0.03701	0.03783	0.03844
11	0.00826	0.01452	0.01932	0.02304	0.02595	0.02824	0.03003	0.03145	0.03257	0.03344
12		0.00694	0.01233	0.01655	0.01989	0.02255	0.02469	0.02640	0.2778	0.02889
13			0.00592	0.01060	0.01433	0.01734	0.01978	0.02177	0.02340	0.02473
14				0.00510	0.00921	0.01254	0.01526	0.01750	0.01935	0.02088
15					0.00444	0.00807	0.01106	0.01353	0.01559	0.01731
16						0.00391	0.00714	0.00983	0.01208	0.01398
17							0.00346	0.00635	0.0879	0.01085
18								0.00309	0.00569	0.00791
19									0.00277	0.00513
20										0.00250

N = number of factors combined, column weights add to 1.0

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