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Decision Support

Simulation of fuzzy multiattribute models for grey relationships

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Abstract

Multiattribute decision making involves tradeoffs among alternative performances over multiple attributes. The accuracy of performance measures are usually assumed to be accurate. Most multiattribute models also assume given values for the relative importance of weights for attributes. However, there is usually some uncertainty involved in both of these model inputs. Outranking multiattribute methods have always provided fuzzy input for performance scores. Many analysts have also recognized that weight estimates also involve some imprecision, either through individual decision maker uncertainty, or through aggregation of diverging group member preferences. Many fuzzy multiattribute models have been proposed, but they have focused on identifying the expected value solution (or extreme solutions). This paper demonstrates how simulation can be used to reflect fuzzy inputs, which allows more complete probabilistic interpretation of model results.

Keywords: Multiple attribute decision making (MADM); Fuzzy sets; Monte Carlo simulation; Grey analysis

1. Introduction

Multiattribute decision making has progressed in a variety of directions throughout the world. Most models are deterministic, to include multiattribute utility theory [8] and AHP [14]. Outranking methods from various schools [9] also support deterministic inputs, although methods such as ELECTRE [12] and PROMETHEE [2] have always supported fuzzy input for alternative performances on attributes.

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Recognition that real life decisions involve high levels of uncertainty is reflected in the development of fuzzy multiattribute models. Fuzzy methods have been widely published in multiattribute decision making [1,3] to include AHP [7]. Uncertain input in the form of rough sets has also been proposed [16]. The method of grey analysis [4] is another approach to reflecting uncertainty in the basic multiattribute model:

$$\text{value}_j = \sum_{i=1}^{K} w_i \times u(x_{ij}), \tag{1}$$

where w_i is the weight of attribute i, K is the number of attributes, and $u(x_{ij})$ is the score of alternative x_j on attribute i.

Grey system theory was developed by Deng [4], based upon the concept that information is sometimes incomplete or unknown. The intent is the same as with factor analysis, cluster analysis, and discriminant analysis, except that those methods often do not work well when sample size is small and sample distribution is unknown [15]. Interval numbers are standardized through norms, which allow transformation of index values through product operations.

This paper addresses the use of Monte Carlo simulation to this model to reflect uncertainty as expressed by fuzzy input. Simulation has been applied to AHP [10], generating random pairwise comparison input values. Our paper differs from past papers in that instead of estimating the expected value or extreme performance of alternatives, simulation offers a more complete understanding of the possible outcomes of alternatives as expressed by fuzzy numbers. The focus is on probability rather than on maximizing expected or extreme values. Both weights and alternative performance scores are allowed to be fuzzy. Both interval and trapezoidal fuzzy input are considered.

2. Grey related analysis

Grey related analysis is a technique that can be applied to both fuzzy and crisp data. Classical grey related analysis is based upon time series data and/or cross-sectional data [11]. This paper extends that approach to a multiattribute decision making context. We present it here as a means to obtain a solution from fuzzy data. Suppose that a multiple attribute decision making problem with interval numbers has m feasible plans $X_1, X_2, \ldots, X_m$, with n indexes. The weight value w_j of index G_j is uncertain, but $w_j \in [c_j, d_j]$, $0 \leqslant c_j \leqslant d_j \leqslant 1$, $j = 1, 2, \ldots, n$, $w_1 + w_2 + \cdots + w_n = 1$, and the index value of jth index G_j of feasible plan X_i is an interval number $[a_{ij}^-, a_{ij}^+]$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$. When $c_j = d_j$, $j = 1, 2, \ldots, n$, the multiple attribute decision making problem with interval numbers is called a multiple attribute decision making problem with interval-valued indexes. When $a_{ij}^- = a_{ij}^+$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$, the multiple attribute decision making problem with interval numbers is called a multiple attribute decision making problem with interval-valued weights. The principle and steps of the grey related analysis method are as follows:

Step 1: Construct decision matrix A with index number of interval numbers

If the index value of jth index G_j of feasible plan X_i is an interval number $[a_{ij}^-, a_{ij}^+]$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$, decision matrix A with index number of interval numbers is defined as the following:

$$A = \begin{bmatrix} [a_{11}^-, a_{11}^+] & [a_{12}^-, a_{12}^+] & \cdots & [a_{1n}^-, a_{1n}^+] \\ [a_{21}^-, a_{21}^+] & [a_{22}^-, a_{22}^+] & \cdots & [a_{2n}^-, a_{2n}^+] \\ \cdots & \cdots & \cdots & \cdots \\ [a_{m1}^-, a_{m1}^+] & [a_{m2}^-, a_{m2}^+] & \cdots & [a_{mn}^-, a_{mn}^+] \end{bmatrix}. \tag{2}$$

Step 2: Transform "contrary index" into positive index

The index is called a positive index if a greater index value is better. The index is called a contrary index if a smaller index value is better. We may transform a contrary index into a positive index if *j*th index G_j is a contrary index

$$[b_{ij}^-, b_{ij}^+] = [-a_{ij}^+, -a_{ij}^-], \quad i = 1, 2, \ldots, m. \tag{3}$$

Without loss of generality, in the following we supposed that all the indexes are "positive indexes".

Step 3: Standardize decision matrix A with index number of interval numbers, obtaining standardizing decision matrix $R = [r_{ij}^-, r_{ij}^+]$

If we mark the column vectors of decision matrix A with interval-valued indexes with$A_1, A_2, \ldots, A_n$, the element of standardizing decision matrix $R = [r_{ij}^-, r_{ij}^+]$ is defined as

$$[r_{ij}^-, r_{ij}^+] = \frac{[a_{ij}^-, a_{ij}^+]}{\|A_j\|}, \quad i = 1, 2, \ldots, m, \; j = 1, 2, \ldots, n. \tag{4}$$

Step 4: Calculate interval number weighted matrix $C = \left(\left[c_{ij}^-, c_{ij}^+\right]\right)_{m\times n}$

The formula for the element of interval number weighted matrix C is

$$\begin{aligned} &C = ([c_{ij}^-, c_{ij}^+])_{m\times n} \\ &[c_{ij}^-, c_{ij}^+] = [c_j, d_j] \cdot [r_{ij}^-, r_{ij}^+], \quad i = 1, 2, \ldots, m, \; j = 1, 2, \ldots, n. \end{aligned} \tag{5}$$

Step 5: Determine reference number sequence

The element of reference number sequence is composed of the optimal weighted interval number index value for every alternative.

$U_0 = \left([u_0^-(1), u_0^+(1)], [u_0^-(2), u_0^+(2)], \ldots, [u_0^-(n), u_0^+(n)]\right)$ is a reference number sequence if $u_0^-(j) = \max_{1\leqslant i\leqslant m} c_{ij}^-$, $u_0^+(j) = \max_{1\leqslant i\leqslant m} c_{ij}^+$, $j = 1, 2, \ldots, n$.

Step 6: Calculate connections between alternatives

First, calculate the connection coefficient $\xi_i(k)$ between the sequence composed of weight interval number standardized index values for every alternative

$U_i = \left([c_{i1}^-, c_{i1}^+], [c_{i2}^-, c_{i2}^+], \ldots, [c_{in}^-, c_{in}^+]\right)$ and the reference number sequence

$U_0 = \left([u_0^-(1), u_0^+(1)], [u_0^-(2), u_0^+(2)], \ldots, [u_0^-(n), u_0^+(n)]\right)$. The formula for $\xi_i(k)$ is

$$\xi_i(k) = \frac{\min_i \min_k |[u_0^-(k), u_0^+(k)] - [c_{ik}^-, c_{ik}^+]| + \rho \max_i \max_k |[u_0^-(k), u_0^+(k)] - [c_{ik}^-, c_{ik}^+]|}{|[u_0^-(k), u_0^+(k)] - [c_{ik}^-, c_{ik}^+]| + \rho \max_i \max_k |[u_0^-(k), u_0^+(k)] - [c_{ik}^-, c_{ik}^+]|}. \tag{6}$$

Here, $\rho \in (0, +\infty)$, and ρ is a resolving coefficient. The smaller ρ is, the greater its resolving power. In general, $\rho \in [0, 1]$. The value of ρ may be changed to reflect the desired degree of resolution.

After calculating $\xi_i(k)$, the connection between the *i*th plan and the reference number sequence is calculated by the following formula:

$$r_i = \frac{1}{n} \cdot \sum_{i=1}^{n} \xi_i(k), \quad i = 1, 2, \ldots, m. \tag{7}$$

Step 7: Determine optimal plan

The feasible plan X_t is optimal if $r_t = \max_{1\leqslant i\leqslant m} r_i$.

3. Demonstration

To demonstrate the methods we present, we draw upon a hiring decision from Royes et al. [13]. This was a fuzzy multicriteria decision problem. The original data was appropriate for its use, but when simulated,

Table 1
Hiring decision input data

Weights	[0.1 0.3 0.3 0.4]	[0.2 0.4 0.5 0.6]	[0 0.1 0.2 0.4]	[0.2 0.3 0.4 0.6]	[0.1 0.2 0.4 0.5]	[0 0.1 0.2 0.4]	[0.2 0.3 0.5 0.6]
Performance	**C1**	**C2**	**C3**	**C4**	**C5**	**C6**	**C7**
Antônio	[0.6 0.7 0.8 0.9]	[0.7 0.8 0.9 1]	[0.2 0.3 0.4 0.5]	[0.4 0.5 0.8 0.9]	[0 0.1 0.3 0.6]	[0.4 0.5 0.7 0.8]	[0.7 0.8 1 1]
Fábio	[0.2 0.3 0.4 0.5]	[0 0.1 0.2 0.3]	[0.6 0.7 0.8 0.9]	[0.2 0.4 0.6 0.7]	[0.2 0.4 0.6 0.9]	[0 0.1 0.2 0.3]	[0 0.1 0.3 0.6]
Alberto	[0.4 0.5 0.6 0.7]	[0.1 0.3 0.7 0.9]	[0.6 0.7 0.8 0.9]	[0.4 0.6 0.7 0.9]	[0.3 0.4 0.8 1]	[0.1 0.3 0.4 0.5]	[0.7 0.8 1 1]
Fernando	[0.8 0.9 1 1]	[0.3 0.4 0.6 0.9]	[0.6 0.7 0.8 0.9]	[0 0.3 0.5 0.8]	[0.2 0.4 0.6 0.8]	[0.4 0.5 0.7 0.9]	[0.3 0.4 0.6 0.8]
Isabel	[0.4 0.6 0.9 1]	[0.6 0.7 0.9 1]	[0.4 0.5 0.6 0.7]	[0.6 0.7 0.9 1]	[0 0.1 0.4 0.6]	[0.4 0.5 0.7 0.9]	[0.4 0.6 0.8 1]
Rafaela	[0.6 0.7 0.8 0.9]	[0.1 0.2 0.3 0.4]	[0.4 0.5 0.6 0.7]	[0.1 0.4 0.6 0.9]	[0 0.1 0.3 0.6]	[0.4 0.5 0.7 0.9]	[0 0.2 0.4 0.7]

Trapezoidal inputs given by: [minimum (value 0), left (value 1), right (value 1), maximum (value 0)].

always yielded one alternative (Isabel) as best. We have expanded the fuzzy data, which is likely when group members are involved in assessing both relative importance of weights, as well as alternative performance over attributes. The problem consists of six applicants for a position, each evaluated over seven attributes. Table 1 gives trapezoidal values for weights as well as for each alternative over each attribute. Attributes are:

C1 experience in the business area
C2 experience in the specific job function
C3 educational background
C4 leadership capacity
C5 adaptability
C6 age
C7 aptitude for teamwork

The overall value for each alternative candidate would be the sum product of weights time performance.

3.1. Interval fuzzy calculation

The grey related method uses interval input. Trapezoidal input as in Table 1 can be converted to interval data using the α-cut technique [5,6] to build membership function the decision measures. This technique based on the resolution principle that a fuzzy set A can be retrieved as a union of its αA_α and the extension principle that:

$$[f(A_1,\ldots,A_r)]_\alpha = f(A_{1\alpha},\ldots,A_{r\alpha}). \tag{8}$$

For any α-level, crisp values of fuzzy variables can be obtained. These crisp values are the points that α intersect with the membership function of that variable, for instance, points A_1 and A_2 in Fig. 1.

Let $\tilde{a} = (a_1, a_2, a_3, a_4)$ be a trapezoidal fuzzy number, which can be depicted in Fig. 2.

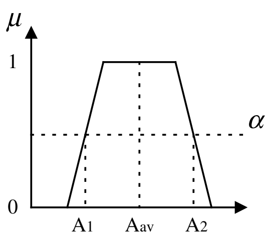

Fig. 1. Crisp values of variable.

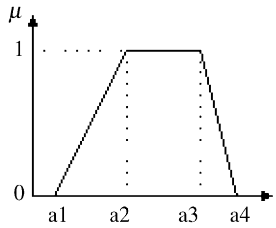

Fig. 2. A trapezoidal fuzzy number.

Using the α-cut technique, $\tilde{a}$ is transformed to an interval number a.

$$a = [\alpha * a_1 + (1 - \alpha) * a_2, \alpha * a_3 + (1 - \alpha) * a_4].$$

In this case, using an α of 0.5, we obtain the data in Table 2.

All of these index values are positive. The next step of the grey related method is to standardize the interval decision matrix. This is necessary if the data is not in a 0–1 range. However, our data is already in that range. Next we need to calculate the interval number weighted matrix C, which consists of the minimum weight times the minimum alternative performance score for each entry as the left element of the interval number, and the maximum weight times the maximum alternative performance score for each entry as the right element of that entry's interval number. The weighted matrix C is shown in Table 3.

Data in Table 3 is rounded simply for display. The convention used in Table 3 was to round the left interval bound down from 0.5, and the right interval up from 0.5. Subsequent calculations were based on the unrounded data.

The next step of the grey related method is to obtain reference number sequences based on the optimal weighted interval number value for every alternative. This is defined as the interval number for each attribute defined as the maximum left interval value over all alternatives, and the maximum right interval value

Table 2
Interval data

Weights	[0.20 0.35]	[0.30 0.55]	[0.05 0.30]	[0.25 0.50]	[0.15 0.45]	[0.05 0.30]	[0.25 0.55]
Performance	**C1**	**C2**	**C3**	**C4**	**C5**	**C6**	**C7**
Antônio	[0.65 0.85]	[0.75 0.95]	[0.25 0.45]	[0.45 0.85]	[0.05 0.45]	[0.45 0.75]	[0.75 1.00]
Fábio	[0.25 0.45]	[0.05 0.25]	[0.65 0.85]	[0.30 0.65]	[0.30 0.75]	[0.05 0.25]	[0.05 0.45]
Alberto	[0.45 0.65]	[0.20 0.80]	[0.65 0.85]	[0.50 0.80]	[0.35 0.90]	[0.20 0.45]	[0.75 1.00]
Fernando	[0.85 1.00]	[0.35 0.75]	[0.65 0.85]	[0.15 0.65]	[0.30 0.70]	[0.45 0.80]	[0.35 0.70]
Isabel	[0.50 0.95]	[0.65 0.95]	[0.45 0.65]	[0.65 0.95]	[0.05 0.50]	[0.45 0.80]	[0.50 0.90]
Rafaela	[0.65 0.85]	[0.15 0.35]	[0.45 0.65]	[0.25 0.75]	[0.05 0.45]	[0.45 0.80]	[0.10 0.55]

Table 3
Weighted matrix C

	Performance						
	C1	**C2**	**C3**	**C4**	**C5**	**C6**	**C7**
Antônio	[0.13 0.30]	[0.22 0.52]	[0.01 0.14]	[0.11 0.43]	[0.01 0.20]	[0.02 0.23]	[0.18 0.55]
Fábio	[0.05 0.16]	[0.01 0.14]	[0.03 0.26]	[0.07 0.33]	[0.04 0.34]	[0.00 0.08]	[0.01 0.25]
Alberto	[0.09 0.23]	[0.06 0.44]	[0.03 0.26]	[0.12 0.40]	[0.05 0.41]	[0.01 0.14]	[0.18 0.55]
Fernando	[0.17 0.35]	[0.10 0.41]	[0.03 0.26]	[0.03 0.33]	[0.04 0.32]	[0.02 0.24]	[0.09 0.39]
Isabel	[0.10 0.33]	[0.19 0.52]	[0.02 0.20]	[0.16 0.48]	[0.01 0.23]	[0.02 0.24]	[0.12 0.50]
Rafaela	[0.13 0.30]	[0.04 0.19]	[0.02 0.20]	[0.06 0.38]	[0.01 0.20]	[0.02 0.24]	[0.02 0.30]

over all alternatives. For **C1**, this would yield the interval number $[0.17, 0.35]$. This reflects the maximum weighted value obtained in the data set for attribute **C1**. Table 4 gives this vector, which reflects the range of value possibilities (entries are not rounded).

Distances are defined as the maximum between each interval value and the extremes generated. Table 5 shows the calculated distances by alternative.

The maximum distance for each alternative to the ideal is identified as the largest distance calculation in each cell of Table 5. These maxima are shown in Table 6.

The reference point is the minimum of all minima and maximum of all maxima distance for each alternative. A reference point is established as the maximum of entries in each column of Table 7. This point has a minimum of 0 and a maximum of 0.3850. Thus the reference point is $[0, 0.385]$.

Next the method calculates the maximum distance between the reference point and each of the weighted matrix C values. The formula for this calculation is formula (6) above. Here we used a ρ value of 0.5. Results by alternative are given in Table 8.

The average of these weighted distances is used as the reference number to order alternatives. These averages reflect how far away each alternative is from the nadir, along with how close they are to the ideal, much as in TOPSIS. This set of numbers indicates that Isabel is the preferred alternative, although Antônio is extremely close, with Alberto and Fernando a little behind. This closeness demonstrates that fuzzy input

Table 4
Reference number vector

	C1	**C2**	**C3**	**C4**	**C5**	**C6**	**C7**
Max(Min)	0.1700	0.2250	0.0325	0.1625	0.0525	0.0225	0.1875
Max(Max)	0.3500	0.5225	0.2550	0.4750	0.4050	0.2400	0.5500

Table 5
Extremes from alternatives to reference number vector

	Distances						
	C1	**C2**	**C3**	**C4**	**C5**	**C6**	**C7**
Antônio	(0.04, 0.0525)	(0, 0)	(0.02, 0.12)	(0.05, 0.05)	(0.045, 0.2025)	(0, 0.015)	(0, 0)
Fábio	(0.12, 0.1925)	(0.21, 0.385)	(0, 0)	(0.0875, 0.15)	(0.0075, 0.0675)	(0.02, 0.165)	(0.175, 0.3025)
Alberto	(0.08, 0.1225)	(0.165, 0.0825)	(0, 0)	(0.0375, 0.075)	(0, 0)	(0.0125, 0.105)	(0, 0)
Fernando	(0, 0)	(0.12, 0.11)	(0, 0)	(0.125, 0.15)	(0.0075, 0.09)	(0, 0)	(0.1, 0.165)
Isabel	(0.07, 0.0175)	(0.03, 0)	(0.01, 0.06)	(0, 0)	(0.045, 0.18)	(0, 0)	(0.0625, 0.055)
Rafaela	(0.04, 0.0525)	(0.18, 0.33)	(0.01, 0.06)	(0.1, 0.1)	(0.045, 0.2025)	(0, 0)	(0.1625, 0.2475)

Table 6
Maximum distances

	Distances						
	C1	**C2**	**C3**	**C4**	**C5**	**C6**	**C7**
Antônio	0.0525	0	0.12	0.05	0.2025	0.015	0
Fábio	0.1925	0.385	0	0.15	0.0675	0.165	0.3025
Alberto	0.1225	0.165	0	0.075	0	0.105	0
Fernando	0	0.12	0	0.15	0.09	0	0.165
Isabel	0.07	0.03	0.06	0	0.18	0	0.0625
Rafaela	0.0525	0.33	0.06	0.1	0.2025	0	0.2475

Table 7
Extreme distances by alternative

	Minimum	Maximum
Antônio	0	0.2025
Fábio	0	0.3850
Alberto	0	0.1650
Fernando	0	0.1650
Isabel	0	0.1800
Rafaela	0	0.3300

Table 8
Weighted distances to reference point

	Distances							Averages
	C1	**C2**	**C3**	**C4**	**C5**	**C6**	**C7**	
Antônio	0.785714	1	0.616000	0.793814	0.487342	0.927711	1	0.801512
Fábio	0.500000	0.333333	1	0.562044	0.740385	0.538462	0.388889	0.580445
Alberto	0.611111	0.538462	1	0.719626	1	0.647059	1	0.788037
Fernando	1	0.616000	1	0.562044	0.681416	1	0.538462	0.771132
Isabel	0.733333	0.865169	0.762376	1	0.516779	1	0.754902	0.804651
Rafaela	0.785714	0.368421	0.762376	0.658120	0.487342	1	0.437500	0.642782

may reflect a case where there is not a clear winner. Simulation provides a tool capable of picking up the probability of each alternative being preferred.

4. Simulation

A simulation model of this decision was generated, with both weights and alternative performance scores over attributes treated as fuzzy numbers between 0 and 1. The simulation was controlled, using ten unique seed values to ensure that the difference in simulation output due to random variation was the same for each alternative.

4.1. Interval fuzzy simulation

The interval fuzzy input shown in Table 2 was modeled first. Two uniform random numbers were drawn for each weight (one for α used to convert the original trapezoidal input), reflecting the proportional distance drawn between the minimum and the maximum. These seven weights were then totaled, and each random weight drawn divided by the total to make the sum of the weights add to one. Uniform random numbers were then drawn for each alternative's performance on each attribute, again reflecting the proportional score. These numbers were not added, as each had the expected random properties directly. The simulation software Crystal Ball was used to replicate each model 1000 times for each random number seed. The software enabled counting the number of times each alternative won. Probabilities given in Table 9 are thus simply the number of times each alternative had the highest value score divided by 1000. This was done ten times, using different seeds. Therefore, mean probabilities and standard deviations (std) are based on 10,000 simulations. The Min and Max entries are the minimum and maximum probabilities in the ten replications shown in table.

Note that alternative Fábio is dominated by alternative Alberto, explaining why Fábio was never selected. Alternative Rafaela also appears to be dominated, although this is not evident by inspection. This

Table 9
Simulated probabilities of winning for uniform fuzzy input

Interval	Antônio	Fábio	Alberto	Fernando	Isabel	Rafaela
seed1234	0.346	0.000	0.206	0.041	0.407	0.000
seed2345	0.375	0.000	0.182	0.035	0.408	0.000
seed3456	0.342	0.000	0.187	0.049	0.422	0.000
seed4567	0.343	0.000	0.193	0.044	0.420	0.000
seed5678	0.360	0.000	0.203	0.048	0.389	0.000
seed6789	0.393	0.000	0.181	0.044	0.382	0.000
seed7890	0.367	0.000	0.182	0.044	0.407	0.000
seed8901	0.336	0.000	0.205	0.035	0.424	0.000
seed9012	0.355	0.000	0.179	0.051	0.415	0.000
seed0123	0.367	0.000	0.171	0.049	0.413	0.000
Min	0.336	0.000	0.171	0.035	0.382	0.000
Mean	0.358	0.000	0.189	0.044	0.409	0.000
Max	0.393	0.000	0.206	0.051	0.424	0.000
Std	0.018	0.000	0.012	0.006	0.014	0.000

demonstrates how the simulation can more accurately identify such cases. Controlled simulation can provide an empirical means of estimating dominated solutions (although admittedly without proof, nor knowing which other alternative or combination of alternative dominated it).

4.2. Trapezoidal fuzzy simulation

The trapezoidal fuzzy input dataset can also be simulated. X is random number ($0 < rn < 1$)

Definition of trapezoid:	$a1$ is left 0 in Fig. 2 $a2$ is left 1 $a3$ is right 1 $a4$ is right 0
Contingent calculation:	J is area of left triangle K is area of rectangle L is area of right triangle Fuzzy sum = left triangle + rectangle + right triangle = 1

M is the area of the left triangle plus the rectangle (for calculation of X value), X is the random number drawn (which is the area)

If $X \leqslant J$:

$$X = a1 + \sqrt{\frac{X \times (a2 - a1) \times (a4 - a3 + a2 - a1)}{J + L}}. \tag{9}$$

If $J \leqslant X \leqslant J + K$:

$$X = a2 + \frac{X - J}{K} \times (a3 - a2). \tag{10}$$

Table 10
Simulated probabilities of winning for trapezoidal fuzzy input

Trapezoidal	Antônio	Fábio	Alberto	Fernando	Isabel	Rafaela
seed1234	0.337	0.000	0.188	0.046	0.429	0.000
seed2345	0.381	0.000	0.168	0.040	0.411	0.000
seed3456	0.346	0.000	0.184	0.041	0.429	0.000
seed4567	0.357	0.000	0.190	0.046	0.407	0.000
seed5678	0.354	0.000	0.210	0.052	0.384	0.000
seed6789	0.381	0.000	0.179	0.046	0.394	0.000
seed7890	0.343	0.000	0.199	0.052	0.406	0.000
seed8901	0.328	0.000	0.201	0.045	0.426	0.000
seed9012	0.353	0.000	0.189	0.048	0.410	0.000
seed0123	0.360	0.000	0.183	0.053	0.404	0.000
Min	0.328	0.000	0.168	0.040	0.384	0.000
Mean	0.354	0.000	0.189	0.047	0.410	0.000
Max	0.381	0.000	0.210	0.053	0.429	0.000
Std	0.017	0.000	0.012	0.004	0.015	0.000

If $J + K \leqslant X$:

$$X = a4 - \sqrt{\frac{(1 - X) \times (a4 - a3) \times (a4 - a3 + a2 - a1)}{J + L}}. \tag{11}$$

Our calculation is based upon drawing a random number reflecting the area (starting on the left ($a1$) as 0, ending on the right ($a4$) as 1), and calculating the distance on the X-axis. The results, following the same procedure as for interval numbers in Table 9, are given in Table 10. The differences are due to drawing two random numbers for the interval data (one for α), while only one random number was needed per parameter in the interval data.

5. Analysis of results

The results for each system were very similar. Differences were tested by t-test of differences in means by alternative. None of these difference tests were significant at the 0.95 level (two-tailed tests). This establishes that no significant difference in interval or trapezoidal fuzzy input was detected (and any that did appear would be a function of random numbers only, as we established an equivalent transformation). A recap of results is given in Table 11.

Isabel still wins, but at a probability just over 0.4. Antônio was second with a probability just over 0.35, followed by Alberto at about 0.19 and Fernando at below 0.05. There was very little overlap among

Table 11
Probabilities obtained

	Antônio	Fábio	Alberto	Fernando	Isabel	Rafaela
Grey-Related	–	–	–	–	X	–
Interval average	*0.358*	*0*	*0.189*	*0.047*	*0.410*	*0*
Interval minimum	0.336	0	0.168	0.040	0.384	0
Interval maximum	0.393	0	0.210	0.053	0.429	0
Trapezoidal average	*0.354*	*0*	*0.189*	*0.044*	*0.409*	*0*
Trapezoidal minimum	0.328	0	0.171	0.035	0.382	0
Trapezoidal maximum	0.381	0	0.206	0.051	0.424	0

alternatives in this example. However, should such overlap exist, the simulation procedure shown would be able to identify it. As it is, Isabel still appears a good choice, but Antônio appears a strong alternative.

6. Conclusions

Multiattribute analysis has long been recognized to involve uncertain input. While some methods have been developed to reflect this uncertainty, for the most part, multiattribute models have been treated as deterministic. Fuzzy multiattribute models (including grey related models) have been popular in the last decade, probably because they reflect uncertainty in inputs, but even those models focus on the expected most preferred choice rather than on the probabilities of alternatives being preferred.

This paper presented simulation of fuzzy multiattribute models, reflecting either interval input or commonly used trapezoidal input. Both weights and alternative performance scores on attributes were allowed to be fuzzy. This is better because uncertainty is represented by a range rather than an expected point value. Thus, simulation can be used to expand analysis to not only identify the most likely best choice, but also to identify related probabilities, even when multiple uncertainties are present that make analytic closed formula solution intractable. Simulation is a useful tool that can be applied to multiattribute models effectively.

References

[1] T. Aouam, S.I. Chang, E.S. Lee, Fuzzy MADM: An outranking method, European Journal of Operational Research 145 (2) (2003) 317–328.
[2] J.P. Brans, Ph. Vincke, A preference ranking organization method: The PROMETHEE method, Management Science 31 (1985) 647–656.
[3] Y.-H. Chang, C.-H. Yeh, A new airline safety index, Transportation Research Part B 38 (2004) 369–383.
[4] J.L. Deng, Control problems of grey systems, Systems and Controls Letters 5 (1982) 288–294.
[5] D. Dubois, H. Prade, Fuzzy Sets and Systems: Theory and Applications, Academic Press, Inc., New York, 1980.
[6] M.E. El-Hawary, Electric Power Applications of Fuzzy Systems, The Institute of Electrical and Electronics Engineers Press, Inc., 1998.
[7] C. Kahraman, U. Cebeci, D. Ruan, Multi-attribute comparison of catering service companies using fuzzy AHP: The case of Turkey, International Journal of Production Economics 87 (2004) 171–184.
[8] R.L. Keeney, H. Raiffa, Decision with Multiple Objectives: Preferences and Value Tradeoffs, Wiley, New York, 1976.
[9] O.I. Larichev, H.M. Moshkovich, ZAPROS: A Method and System for Ordering Multiattribute Alternatives on the Base of a Decision-maker's Preferences, All-Union Research Institute for System Studies, Moscow, 1991.
[10] R.R. Levary, K. Wan, A simulation approach for handling uncertainty in the analytic hierarchy process, European Journal of Operational Research 106 (1998) 116–122.
[11] S. Liu, B. Guo, Y. Dang, Grey System Theory and Applications, Scientific Press, Beijing, 1999.
[12] B. Roy, ELECTRE III: Un algorithme de classement fonde sur une representation floue des preferences en presence de criteres multiple, Cahiers du Centre Etudes Recherche Operationelle 20 (1978) 3–24.
[13] G.F. Royes, R.C. Bastos, G.F. Royes, Applicants' selection applying a fuzzy multicriteria CBR methodology, Journal of Intelligent and Fuzzy Systems 14 (4) (2003) 167–180.
[14] T.L. Saaty, How to make a decision: The analytic hierarchy process, European Journal of Operational Research 48 (1990) 426–447.
[15] R.-T. Wang, C.-T. Ho, C.-M. Feng, Y.-K. Yang, A comparative analysis of the operational performance of Taiwan's major airports, Journal of Air Transport Management 10 (2004) 353–360.
[16] K. Zaras, Rough approximation of a preference relation by a multi-attribute dominance for deterministic, stochastic and fuzzy decision problems, European Journal of Operational Research 159 (2004) 196–206.